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THE PRINCIPLES OF THE DYNAMIC THEORY

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Extensions of current physical theories required by the Dynamic Theory are displayed. In these extensions new field quantities appear that become important for systems with varying mass density.

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I. INTRODUCTION

The objective of this report is to present the principles of a theory which was developed partly during thesis research and partly during a research project sponsored by the Naval Academy Research Council. That portion of the theory developed during thesis research at the Naval Postgraduate School is contained in the thesis titled "On a Possible Formulation of Particle Dynamics in Terms of Thermodynamic Conceptualizations and the Role of Entropy in it." In order that this report may present a complete picture of the theory some of the material presented in the thesis is developed in sections II.A and II.B.

During any theorization the philosophy of the theorist plays such an important role that an attempt to understand the theory is aided by a knowledge of this philosophy. Therefore this report includes not only the mathematical development but also the philosophical basis upon which the theory is based.

Einstein, in the Special Theory of Relativity, adopted the position that the constancy of the speed of light forces a modification of Newton's Dynamic Law. This modification implies that all forces have the same limiting velocity, namely, the speed of light. There exists an abundance of theoretical and experimental evidence that the speed of light becomes the limiting velocity whenever electromagnetic forces are involved. But what of the other forces, such as gravitional? Should they also have the same limiting velocity? Though we have had reports of the detection of gravitational waves we have no experimental determination of the speed of a gravitational wave. Therefore, it appears that this modification should have some additional justification.

To further illustrate this conclusion suppose we consider an analogy, which may not hold in the strictest sense, that will show the adopted point of view. A river flowing toward the sea carries energy with it. The speed with which this energy can move from one point to another is the velocity of the river's current. The river produces a force on a boat tied up to a pier on the river. When the boat is set adrift this force accelerates the boat. However, the maximum velocity to which the river can accelerate the boat is the current velocity. This is the velocity with which the energy of the river can propagate.

From this point of view the speed of light, being the propagation velocity of electromagnetic energy must be the limiting velocity associated with electromagnetic forces. Certainly nature would be much simplier if all forces have the same limiting velocity. Yet without some experimental evidence of the propagation velocity of gravitational energy there seems to be no *a priori* reason that gravitational forces should be subjected to the same limiting velocity other than arguments of simplicity.

Is nature symmetrical in time? Does everything run backwards in time as well as forward? Obviously not every process in nature will run backwards, yet the equations of motion in Newtonian and relativistic mechanics are time symmetrical. Certainly there are special cases in nature where time symmetry appears and time symmetrical equations of motion should be used. However, these equations are limited to those special cases for the universal application of time symmetrical equations could not describe a time asymmetrical nature.

There are several different branches of physics currently being used, such as thermodynamics, Newtonian and relativistic mechanics and quantum mechanics. Each of these branches has its own basic assumptions or postulates. There has been a great deal of effort by physicists over the last seventy years to somehow merge these branches into a unified theory. This unification can now be done.

Think of a group of tall, ancient redwood trees. Each tree is strong, tall, and above the ground is very distinct from the others. However, we know that if we dig down below the ground we will find that these trees grow from the same root system. This is similar to the philosophy adopted here concerning the different branches of physics. If we obtain a more fundamental level we should find that the different branches are merely special cases of the more general laws of nature. The prior attempts of unification seem to be trying to tie the trees together at the top rather than down at the root level.

Suppose we adopt the viewpoint that there should be a single set of laws, or postulates, which will yield the different branches of physics. How might we begin to determine what they are? In order to illustrate the procedure adopted within this theory consider the directions that a native Ozarkian gave to a stranger who was trying to find a certain fishing hole. The directions went something like this; "See yonder road going down that holler? Well, go down thar 'bout 5 mile and you'll come to a fork in the road. Take the right hand ford. Now that's the wrong one but you take it anyways. After you've gone a piece you'll come to a log across the road. Now you know you're on the wrong road. So go back and take the left-hand fork. You can't miss it."

How does this bit of Ozark hill wit help determine a more fundamental set of physical laws? Recall that Newtonian mechanics fails to describe events involving high velocities, relativistic mechanics fails to describe the atom, and gravitational effects have resisted quantization. If we view these as logs and follow the Ozarkian's directions we must retrace our steps and seek another approach rather than attempting to chop the log up and continue to push forward up one of these roads.

We find then that thermodynamics is the one branch which does not appear to have a log somewhere along the way. Here we find the classical thermodynamic laws very general, particularly Coratheodory's statement of the second law. If we follow the directions the thermodynamic laws seem to be the place to look for the fork in the road where we might hope to take a different route.

In mechanics we talk of equations of motion, field equations, and geometry while in thermodynamics we speak of equations of state and equilibrium. If we adopt a generalization of the classical thermodynamic laws how could we obtain the equations we are familiar with in mechanics? The crucial point is how to obtain geometry from these general laws for if we have a geometrical description we can use established procedures to obtain equations of motion and field equations.

Geometry may be obtained from a quadratic form and we know from thermodynamics that the stability conditions appear as a quadratic form and therefore the stability conditions should yield a natural geometry based upon laws generalized from the classical thermodynamic laws.

Now obviously such a generalization will yield classical thermodynamics if only thermodynamic variables are considered. What we need to find out is whether or not these laws can also yield familiar mechanical laws.

Chapter II of this report covers this question beginning with the adopted laws as they would be applied to a mechanical system. Next the stability condition quadratic form is derived and the nature of the resulting geometry is determined. Once the geometry has been determined the field equations and quantization follow from the geometry and the laws using the previous work of Weyl and London.

Once having established the unifying effect of the theory an immediate question appears. What about new predictions, or results? If when we hold the mechanical variables fixed we get classical thermodynamics and when we hold the thermodynamic variables fixed we obtain the mechanical theories, what do we get if we have a system which must be described in terms of both thermodynamic and mechanical variables? Chapter III presents some of the immediate results which may be obtained for such a system.

II. UNIFYING EFFECT OF THE DYNAMIC THEORY

A. General Laws

In the following development physical concepts are necessary, as are symbols for these concepts. Because this development will merge certain thermodynamic conceptualizations into mechanics, a notational dilemma must be faced. On the one hand it is desired to preserve the thermodynamic conceptualization by using familiar symbols from that theory. On the other hand it is really mechanical systems for which a description is sought. The formulism then looks either like thermodynamics with familiar thermodynamic quantities replaced by mechanical quantities, or it looks like mechanics into which thermodynamic quantities intruded. In either case there is danger of confusion. One could evade the dilemma by choosing entirely different symbols for the variables of the theory. But then the whole takes an artificially abstract character. Since the purpose of this formulation is to bring out the power of the thermodynamic conceptualization it was decided to use the suggestiveness of the thermodynamic or mechanical symbols whenever convenient and the reader is asked to keep an open mind and not make premature associations with the symbols used.

1. First Law

The concept of conservation of energy is fundamental to all branches of physics and therefore represents a logical beginning for a generalized theory. Therefore, in terms of generalized coordinates the notion of work, or mechanical energy, is considered linear forms of the type

$$đW = F_i(q^1, \ldots, q^n, \dot{q}^1, \ldots, \dot{q}^n)dq^i; \; (i = 1, 2, \ldots, n)$$

Where the forces F_i may be functions of the velocities ($dq^i/dt \equiv \dot{q}^i$) as well as the coordinates q^i and the summation convention is used.

The line integral $\int_C F_i\, dq^i$ then represents the work done along the path C by the generalized forces.

A system may acquire energy other than mechanical, such energy acquisition is denoted $đQ$.

The system energy, which represents the energy possessed by the system, is considered to be

$$\mathcal{U}(q^1, \ldots, q^n, \dot{q}^1, \ldots q^n).$$

$d\mathcal{U}$ will be assumed to be a perfect differential.

With these concepts then the generalized law of conservation of energy has the form

$$\begin{aligned} đQ &\equiv d\mathcal{U} - đ\mathcal{W} \\ &= d\mathcal{U} - F_i\, dq^i;\ (i = 1, \ldots, n) \end{aligned} \tag{1}$$

Positive $đQ$ is taken as energy added to the system by means other than mechanical and F_i is taken as the component of the generalized force acting on the system.

In an infinitesimal transformation, the first law is equivalent to the statement that the differential

$$d\mathcal{U} = đQ + Fidq^i$$

is exact. That is, there exists a function $\mathcal{U}$ whose differential is $d\mathcal{U}$; or the integral $\int d\mathcal{U}$ is independent of the path of the integration and depends only on the limits of integration. This condition is not shared by $đQ$ or $đ\mathcal{W}$.

Since this statement of the generalized first law is consistent with the first law of thermodynamics and it is desired to derive the equations of motion for a mechanical system from the generalized laws, all thermodynamic coordinates will be held fixed. To simplify the initial development only one positional coordinate will be used and the extension to larger dimensional systems made at an appropriate later time.

To explore some of the consequences of the exactness of $d\mathcal{U}$ consider a system whose variables are F, $\dot{q}$ and q. The existence of the state function $\mathcal{U}$, or an equation of state, means that any pair of these three parameters may be chosen to be the independent variables that completely specify the system. For example consider $\mathcal{U} = \mathcal{U}(F, q)$, then

$$d\mathcal{U} = \left(\frac{\partial \mathcal{U}}{\partial F}\right)_q dF + \left(\frac{\partial \mathcal{U}}{\partial q}\right)_F dq,$$

The requirement that $d\mathcal{U}$ be exact immediately leads to the result

$$\frac{\partial}{\partial q}\left[\left(\frac{\partial \mathcal{U}}{\partial F}\right)_q\right]_F = \frac{\partial}{\partial F}\left[\left(\frac{\partial \mathcal{U}}{\partial q}\right)_F\right]_q .$$

The "energy capacity" of a system at the position q with dq = o may be defined as

$$C_q \equiv \left(\frac{\Delta Q}{\Delta \dot{q}}\right)_q = \left(\frac{\partial \mathcal{U}}{\partial \dot{q}}\right)_q ,$$

while the "energy capacity" of a system under a constant force is defined as

$$C_F \equiv \left(\frac{\Delta Q}{\Delta q}\right)_F = \left(\frac{\partial \mathcal{U}}{\partial q}\right) - F\left(\frac{\partial q}{\partial q}\right)_F$$

2. Second Law

There are processes which satisfy the first law but which are not observed in nature. The purpose of the dynamical second law is to incorporate such experimental facts into the model of dynamics.

The statement of the second law is made using the axiomatic statement provided by the Greek mathematician Caratheodory, who presented an axiomatic development of the second law of thermodynamics which may be applied to a system of any number of variables. The second law may then be stated as:

In the neighborhood (however close)
of any equilibrium state of a system
of any number of dynamic coordinates,

there exists states that cannot
be reached by reversible Q -
conservative ($\bar{d}Q = o$) processes.

When the variables are thermodynamic variables the Q-conservative processes are known as adiabatic processes.

A reversible process is one that is performed in such a way that, at the conclusion of the process, both the system and the local surroundings may be restored to their initial states, without producing any change in the rest of the universe.

Consider a system whose independent coordinates are a generalized displacement denoted q, a generalized velocity $\dot{q}$ (with $\dot{q} \equiv dq/dt$), and a generalized force F. It can be shown that the Q-conservative curve comprising all equilibrium states accessible from the initial state, i, may be expressed by

$$\sigma = \sigma(\dot{q}, q) = \text{constant}$$

where σ represents some as yet undetermined function. Curves corresponding to other initial states would be represented by different values of the constant.

Reversible Q-conservative curves cannot intersect, for if they did it would be possible, as shown in Figure 1, to proceed from an initial equilibrium state i, at the point of intersection, to two different final states f_1 and f_2, having the same q, along reversible Q-conservative paths, which is not allowed by the second law.

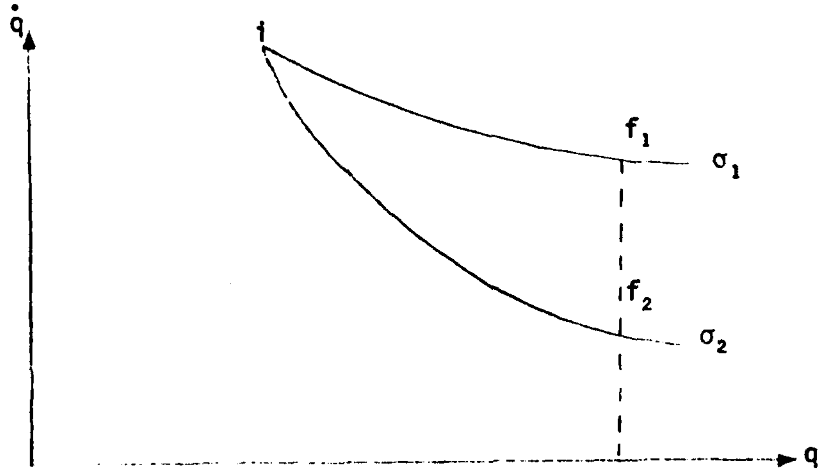

Figure 1. If two reversible Q-conservative curves could intersect, it would be possible to violate the second law by performing the cycle i, f_1, f_2, i.

When the system can be described with only two independent variables, such as on the Q-conservative curve, then if these variables are $\dot{q}$ and q, and F is a generalized force,

$$đQ = dU - Fdq.$$

Regarding $U = U(\dot{q},q)$ then

$$đQ = \left(\frac{\partial U}{\partial \dot{q}}\right)_q d\dot{q} + \left[\left(\frac{\partial U}{\partial q}\right)_{\dot{q}} - F\right] dq,$$

where $\left(\frac{\partial U}{\partial \dot{q}}\right)_q$, F, and $\left(\frac{\partial U}{\partial q}\right)_{\dot{q}}$ are functions of $\dot{q}$ and q.

A Q - conservative process for this system is

$$\left(\frac{\partial U}{\partial \dot{q}}\right)_q d\dot{q} + \left[\left(\frac{\partial U}{\partial q}\right)_{\dot{q}} - F\right] dq = o \qquad (2)$$

Solving for $d\dot{q}/dq$ yields

$$\frac{d\dot{q}}{dq} = \frac{-\left[\left(\frac{\partial U}{\partial q}\right)_{\dot{q}} - F\right]}{\left(\frac{\partial U}{\partial \dot{q}}\right)_q}$$

The right hand member is a function of $\dot{q}$ and q, and therefore the derivative $d\dot{q}/dq$, representing the slope of a Q-conservative curve on a $(\dot{q}, q)$ diagram, is known at all points. Equation (2) has therefore a solution consisting of a family of curves, see Figure 2, and the curve through any one point may be written

$$\sigma = \sigma(\dot{q}, q) = \text{constant}$$

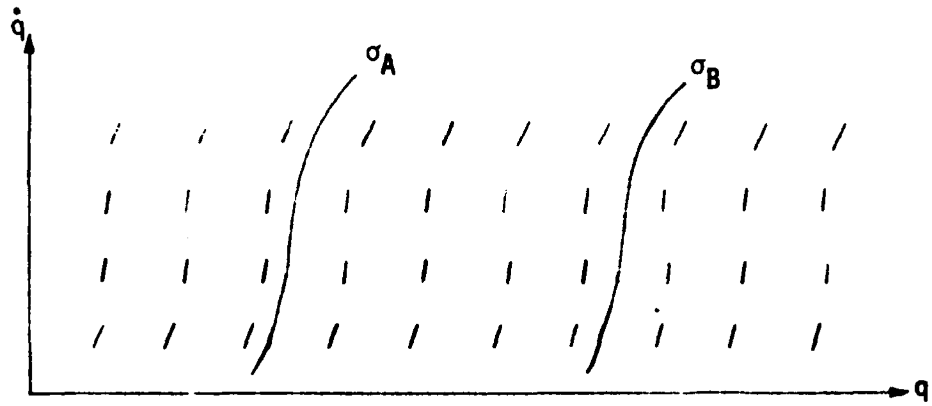

Figure 2. The first law, through equation (2) fills the $(\dot{q}, q)$ space with slopes specified at each point. The σ curves represent the solution curves whose tangents are the required slopes. The second law requires that these curves do not intersect.

A set of curves is obtained when different values are assigned to the constant. The existence of the family of curves $\sigma(\dot{q}, q)$ = constant, generated by equation (2), representing reversible Q-conservative processes, follows from the fact that there are only two independent variables and not from any law of physics. Thus it can be seen that the first law may be satisfied by any of these σ = constant curves. The axiom requires that these curves do not intersect. Therefore the axiom, together with the first law, leads to the conclusion that: through any arbitrary initial-state point, all reversible Q-conservative processes lie on a curve, and Q-conservative curves through other initial states determine a family of non-intersecting curves.

To see the results of this conclusion consider a system whose coordinates are the generalized velocity $\dot{q}$, the generalized displacement q and the generalized force F. The first law is

$$đQ = dU - Fdq$$

where U and F are functions of $\dot{q}$ and q. Since the $(\dot{q}, q)$ surface is subdivided into a family of non-intersecting Q-conservative curves

$$\sigma(\dot{q}, q) = \text{constant}$$

where the constant can take on various values σ_1, σ_2, ... any point in the surface may be determined by specifying the value of σ along with q so that U, as well as F, may be regarded as functions of σ and q. Then

$$dU = \left(\frac{\partial U}{\partial \sigma}\right)_q d\sigma + \left(\frac{\partial U}{\partial q}\right)_\sigma dq$$

and

$$đQ = \left(\frac{\partial U}{\partial \sigma}\right)_q d\sigma + \left[\left(\frac{\partial U}{\partial q}\right)_\sigma - F\right] dq$$

Since σ and q are independent variables this equation must be true for all values of $d\sigma$ and dq.

Suppose $d\sigma = 0$ and $dq \neq 0$. The provision that $d\sigma = 0$ is the provision for a Q-conservative process in which $đQ = 0$. Therefore, the coefficient of dq must vanish. Then, in order for σ and q to be independent and for $đQ$ to be zero when $d\sigma$ is zero, the equation for $đQ$ must reduce to

$$đQ = \left(\frac{\partial U}{\partial \sigma}\right)_q d\sigma,$$

with

$$\left(\frac{\partial U}{\partial q}\right)_\sigma = F.$$

Defining a function λ by

$$\lambda \equiv \left(\frac{\partial U}{\partial \sigma}\right)_q,$$

then

$$đQ = \lambda \, d\sigma ,$$

where

$$\lambda = \lambda(\sigma,q).$$

Now, in general, an infinitesimal of the type

$$Pdx + Qdy + Rdz + \dots ,$$

known as a linear differential form, or a Pfaffian expression, when it involves three or more independent variables, does not admit of an integrating factor. It is only because of the existence of the axiom that the differential form for $đQ$ referring to a physical system of any number of independent coordinates possess an integrating factor.

Two infinitesimally neighboring reversible Q-conservative curves are shown in Figure 3. One curve is characterized by a constant value of the function σ_A, and the other by a slightly different value $\sigma_A + d\sigma = \sigma_B$. In any process represented by a displacement along either of the two Q-conservative curves $đQ = 0$. When a reversible process connects the two Q-conservative curves energy $đQ = \lambda d\sigma$ is transferred.

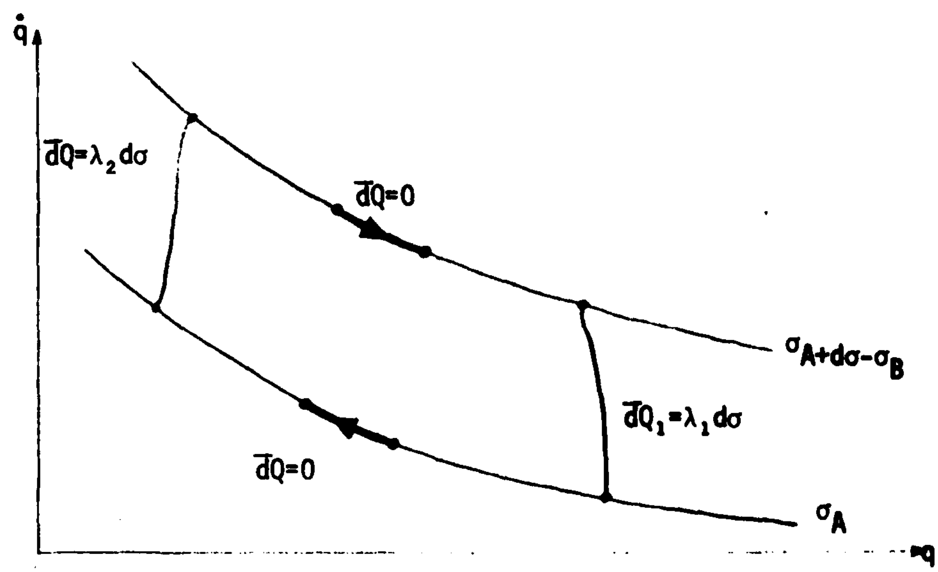

Figure 3. Two reversible Q-conservative curves, infinitesimally close, when the process is represented by a curve connecting the Q-conservative curves, energy $đQ = \lambda d\sigma$ is transferred.

The various infinitesimal processes that may be chosen to connect the two neighboring reversible Q-conservative curves, shown in Figure 3, involve the same change of σ but take place at different λ. In general λ is a function of $\dot{q}$ and q. However it is obvious that λ may be expressed as a function of σ and $\dot{q}$. To find the velocity dependence of λ consider two systems, one and two, such that in the first system there are two independent coordinates $\dot{q}$ and q and the Q-conservative curves are specified by different values of the function σ of $\dot{q}$ and q. When energy $đQ$ is transferred, σ changes by $d\sigma$ and $đQ = \lambda d\sigma$ where λ is a function of σ and g.

The second system has two independent coordinates $\dot{q}$, and $\hat{q}$ and the Q-conservative curves are specified by different values of the function $\hat{\sigma}$ of $\dot{q}$ and $\hat{q}$. When $\hat{đQ}$ is transferred, $\hat{\sigma}$ changes by $d\hat{\sigma}$ and $\hat{đQ} = \hat{\lambda} d\hat{\sigma}$ where $\hat{\lambda}$ is a function of $\hat{\sigma}$ and $\dot{q}$.

The two systems are related through the coordinate $\dot{q}$ in that both systems make up a composite system in which there are three independent coordinates $\dot{q}$, q, and $\hat{q}$ and the Q-conservative curves are specified by different values of the function σ_c of these independent variables.

Since $\sigma = \sigma(\dot{q},q)$ and $\hat{\sigma} = \hat{\sigma}(\dot{q},q)$, using the equations for σ and $\hat{\sigma}$, σ_c may be regarded as a function of $\dot{q}$, σ and $\hat{\sigma}$.

For an infinitesimal process between two neighboring Q-conservative surfaces specified by σ_c and $\sigma_c + d\sigma_c$, the energy transferred is $đQ_c = \lambda_c d\sigma_c$ where λ_c is also a function of $\dot{q}$, σ, and $\hat{\sigma}$. Then

$$d\sigma_c = \frac{\partial \sigma_c}{\partial \dot{q}} d\dot{q} + \frac{\partial \sigma_c}{\partial \sigma} d\sigma + \frac{\partial \sigma_c}{\partial \hat{\sigma}} d\hat{\sigma} \qquad (3)$$

Now suppose that in a process there is a transfer of energy $đQ_c$ between the composite system and an external reservoir with energies $đQ$ and

$\bar{d}\hat{Q}$ being transferred, respectively, to the first and second systems, then

$$\bar{d}Q_c = \bar{d}Q + \bar{d}\hat{Q}$$

and

$$\lambda_c \, d\sigma_c = \lambda d\sigma + \hat{\lambda} d\hat{\sigma} \ ,$$

or

$$d\sigma_c = \frac{\lambda}{\lambda_c} d\sigma + \frac{\hat{\lambda}}{\lambda_c} d\hat{\sigma} \ . \tag{4}$$

Comparing equations (3) and (4) for $d\sigma_c$ then

$$\frac{\partial \sigma_c}{\partial \dot{q}} = 0.$$

Therefore σ_c does not depend on $\dot{q}$, but only on σ and $\hat{\sigma}$. That is

$$\sigma_c = \sigma_c(\sigma, \hat{\sigma}) \ .$$

Again comparing the two expressions for $d\sigma_c$

$$\frac{\lambda}{\lambda_c} = \frac{d\sigma_c}{d\sigma} \text{ and } \frac{\hat{\lambda}}{\lambda_c} = \frac{d\sigma_c}{d\hat{\sigma}} \ ,$$

therefore the two ratios λ/λ_c and $\hat{\lambda}/\lambda_c$ are also independent of $\dot{q}$, q and $\hat{q}$. These two ratios depend only on the σ's, but each separate λ must depend on the velocity as well (for example, if λ depended only on σ and on nothing else, the $\bar{d}Q = \lambda d\sigma$ would equal $f(\sigma)\, d\sigma$ which is an exact differential). In order for each λ to depend on the velocity and at the same for the ratios of the λ's to depend only on the σ's, the λ's must have the following structure:

$$\begin{aligned} \lambda &= \phi(\dot{q}) \ f(\sigma), \\ \hat{\lambda} &= \phi(\dot{q}) \ \hat{f}(\hat{\sigma}), \end{aligned} \tag{5}$$

and

$$\lambda_c = \phi(\dot{q}) \ g(\sigma, \hat{\sigma}).$$

(The quantity λ cannot contain q, nor can $\hat{\lambda}$ contain $\hat{q}$, since λ/λ_c and $\hat{\lambda}/\lambda_c$ must be functions of the σ's only.)

Referring now only to the first system as representative of any system of any number of independent coordinates, the transferred energy is, from equations (5),

$$\bar{d}Q = \phi(\dot{q})\ f(\sigma)\ d\sigma$$

Since $f(\sigma)d\sigma$ is an exact differential, the quantity $1/\phi(\dot{q})$ is an integrating factor for $\bar{d}Q$. It is an extraordinary circumstance that not only does an integrating factor exist for the $\bar{d}Q$ of any system, but this integrating factor is a function of velocity only and is the same function for all systems.

The fact that a system of two independent variables has a $\bar{d}Q$ which always admits an integrating factor regardless of the axiom is interesting, but its importance in physics is not established until it is shown that the integrating factor is a function of velocity only and that it is the same function for all systems.

3. The Absolute Velocity

The universal character of $\phi(\dot{q})$ makes it possible to define an absolute velocity. Consider a system of two independent variables q and $\dot{q}$, for which two constant velocity curves and Q-conservative curves are shown in Figure 4. Suppose there is a constant velocity transfer of energy Q between the system and an external reservoir at the velocity $\dot{q}$, from a state b, on a Q-conservative curve characterized by the value σ_1, to another state c, on another Q-conservative curve specified by σ_2. Then since

$$\bar{d}Q = \phi(\dot{q})f(\sigma)d\sigma,$$

it is seen that

$$\Delta Q = \phi(\dot{q}) \int_{\sigma_1}^{\sigma_2} f(\sigma)d\sigma \text{ at constant } \dot{q}.$$

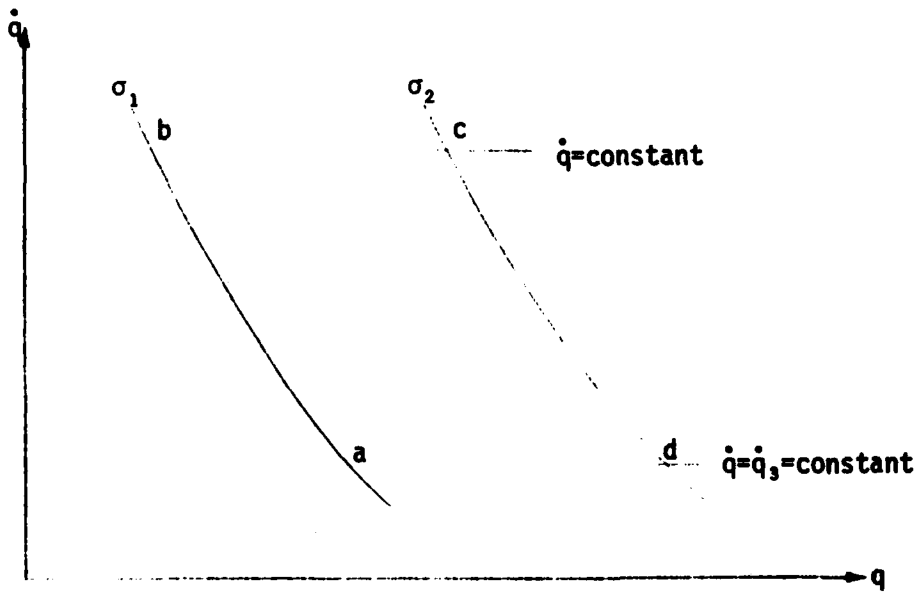

Figure 4. Two constant velocity energy transfers, Q at $\dot{q}$ from b to c and Q_3 at $\dot{q}_3$ from a to to d, between the same two conservative curves σ_1 and σ_2.

For any constant velocity process between two other points a to d, at a velocity $\dot{q}_3$ between the same two Q-conservative curves the energy transferred is $\Delta Q(\dot{q}_3) = \Delta Q_3 = \phi(\dot{q}_3) \int_{\sigma_1}^{\sigma_2} f(\sigma)\, d\sigma$ at constant $\dot{q}_3$

Taking the ratio of

$$\frac{\Delta Q}{\Delta Q_3} = \frac{\phi(\dot{q})}{\phi(\dot{q}_3)} = \frac{\text{a function of the velocity at which } \Delta Q \text{ is transferred}}{\text{same function of velocity at which } \Delta Q_3 \text{ is transferred}}$$

Then the ratio of these two functions is defined by

$$\frac{\phi(\dot{q})}{\phi(\dot{q}_3)} = \frac{\Delta Q(\text{between } \sigma_1 \text{ and } \sigma_2 \text{ at } \dot{q})}{\Delta Q_3\ (\text{between } \sigma_1 \text{ and } \sigma_2 \text{ at } \dot{q}_3)}$$

or

$$\Delta Q = [\frac{\Delta Q_3}{\phi(\dot{q}_3)}]\ \phi(\dot{q}),$$

by choosing some appropriate velocity $\dot{q}_3$ then it follows that the energy transferred at constant velocity between two given Q-conservative curves decreases as $\phi(\dot{q})$ decreases, or the smaller the value of Q the lower the corresponding value of $\phi(\dot{q})$. When ΔQ is zero $\phi(\dot{q})$ is also zero. The corresponding velocity $\dot{q}_0$ such that $\phi(\dot{q}_0)$ is zero is the "absolute velocity". Therefore, if a system undergoes a constant velocity process between two Q-conservative curves without an exchange of energy, the velocity at which this takes place is called the absolute velocity.

4. The Concept of Entropy

In a system of two independent variables, all states accessible from a given initial state by reversible Q-conservative processes lie on a $\sigma(\dot{q},q)$ curve. The entire $(\dot{q},q)$ space may be conceived as being filled by many non-intersecting curves of this kind, each corresponding to a different value of σ. In a reversible non Q-conservative process involving a transfer of energy $\bar{d}Q$, a system in a state represented by a point lying on a surface σ will change until its state point lies on another surface $\sigma + d\sigma$. Then

$$\bar{d}Q = \lambda d\sigma,$$

where $1/\lambda$, the integrating factor of $\bar{d}Q$, is given by

$$\lambda = \phi(\dot{q})f(\sigma),$$

and therefore

$$\bar{d}Q = \phi(\dot{q})f(\sigma)d\sigma$$

or

$$\frac{\bar{d}Q}{\phi(\dot{q})} = f(\sigma)d\sigma\ .$$

Since σ is an actual function of $\dot{q}$ and q the right-hand member is an exact differential, which may be denoted by dS; and

$$dS = \frac{\bar{d}Q}{\phi(\dot{q})} ,$$

where S is the mechanical entropy of the system and the process is a reversible one.

The dynamical second law may be used to prove equivalent of Clausius' theorem, which is stated here without proof.

Theorem: In any cyclic transformation throughout which the velocity is defined, the following inequality holds:

$$\oint \frac{\bar{d}Q}{\phi(\dot{q})} \leq 0 ,$$

where the integral extends over one cycle of the transformation. The equality holds if the cyclic transformation is reversible. Then for an arbitrary transformation

$$\int_A^B \frac{\bar{d}Q}{\phi(\dot{q})} \leq S(B) - S(A),$$

with the equality holding if the transformation is reversible. The proof of this statement may be seen by letting R and I denote respectively any reversible and any irreversible path joining A to B, as shown in Figure 5.

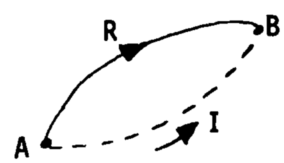

Figure 5

For path R the assertion holds by definition of S. Now consider the cyclic transformation made up of I plus the reverse of R. From Clausius' theorem

$$\int_I \frac{\bar{d}Q}{\phi} - \int_R \frac{\bar{d}Q}{\phi} \leq 0 ,$$

or

$$\int_I \frac{\bar{d}Q}{\phi} \leq \int_R \frac{\bar{d}Q}{\phi} \equiv S(B) - S(A).$$

Another result of the dynamical second law is that the mechanical entropy of an isolated ($\bar{d}Q = 0$) system never decreases. This can be seen since an isolated system cannot exchange energy with the external world since $\bar{d}Q = 0$ for any transformation. Then by the previous property of the entropy,

$$S(B) - S(A) \geq 0$$

where the equality holds if the transformation is reversible.

One consequence of the second law is that of all the possible transformations from one state A to another state B the one defined as the change in the entropy is the one for which the integral

$$I \equiv \int_A^B \frac{\bar{d}Q}{\phi}$$

is a maximum. Thus

$$S(B) - S(A) \equiv \text{maximum } I = \max \int_A^B \left(\frac{1}{\phi} \frac{\bar{d}Q}{d\tau}\right) d\tau,$$

where τ is a parameter which indicates position along the path from A to B,

or $$S(B) - S(A) \quad \max \int_A^B \left(\frac{1}{\phi} \frac{dU}{d\tau} - \frac{F}{\phi} \frac{dq}{d\tau}\right) d\tau;$$

If

$$U = U(\tau, q, \dot{q}, \frac{d\dot{q}}{d\tau})$$

where $\dot{q}^i = dq^i/dt$, then the change in the entropy is given by the integral

$$\Delta S = \int_A^B \left(\frac{1}{\phi} \frac{dU}{d\tau} - \frac{F}{\phi} \frac{dq}{d\tau}\right) d\tau.$$

The $\dot{q}$ and q which maximize ΔS will be denoted as $\dot{x}$ and x then, with

$$U = U(x, \dot{x})$$
$$F_i = F_i(x, \dot{x})$$
$$\phi = \phi(\dot{x})$$

the x and $\dot{x}$ are given by the solution of the system of equations

$$\frac{d}{d\tau}\left(\frac{\partial G}{\partial x'}\right) - \frac{\partial G}{\partial x} = 0$$

$$\frac{d}{d\tau}\left(\frac{\partial G}{\partial \dot{x}'}\right) - \frac{\partial G}{\partial \dot{x}} = 0$$

where

$$G = \left(\frac{1}{\phi}\right)\left[\frac{\partial U}{\partial \tau} - F_i \frac{dx}{d\tau}\right] \text{ and } x' = \frac{dx}{d\tau} \text{ and } \dot{x}' = \frac{d\dot{x}}{d\tau} .$$

Thus the dynamical second law provides an answer to the question that is not contained within the scope of the first law: In what direction does a process take place? The answer is that a process always takes place in such a direction as to cause an increase of the mechanical entropy in the universe. In the case of an isolated system, it is the entropy of the system that tends to increase. To find out, therefore, the equilibrium state of an isolated one dimensional system, it is necessary merely to express the entropy as a function of q and $\dot{q}$ and to apply the usual rules of calculus to render the function a maximum. When the system is not isolated there are other entropy changes to be taken into account.

5. Third Law

The dynamical second law enables the mechanical entropy of a system to be defined up to an arbitrary additive constant. The definition depends on the existence of a reversible transformation connecting an arbitrarily chosen reference state 0 to the state under consideration. Such a reversible transformation always exists if both 0 and A lie on one sheet of the equation of the state surface. If two different systems are considered the equation of the state surface may consist of several disjoint sheets. In such cases the kind of reversible path previously mentioned may not exist. Therefore the second law does not uniquely determine the difference in entropy of two states A and B, if A defines a state of one system and B the state of another. For this determination a dynamical third law is needed. The dynamical third law may be stated, "The mechanical entropy of a system at the absolute velocity is a universal constant, which may be taken to be zero." In the case of a purely thermodynamic system the absolute quantity is the absolute zero temperature, while for a mechanical system the absolute quantity is the absolute velocity.

The dynamical third law implies that any energy capacity of a system must vanish at the absolute velocity. To see this, let R be any reversible path connecting a state of the system at the absolute velocity $\dot{q}_0$ to the state A, whose entropy is to be found. Let $C_R(\dot{q})$ be the energy capacity of the system along the path R. Then, by the second law,

$$\mathcal{S}(A) = \int_{\dot{q}_0}^{\dot{q}_A} C_R(\dot{q}) \frac{d\dot{q}}{\phi(\dot{q})}.$$

But according to the third law,

$$\mathcal{S}(A) \to 0.$$
$$\dot{q}_A \to \dot{q}_0$$

Hence it follows that

$$C_R(\dot{q}) \to 0$$
$$\dot{q} \to \dot{q}_0.$$

In particular, C_R may be C_q or C_F.

B. Equilibrium and Stability Conditions

The three generalized laws have been formulated and a few results of these laws have been seen. The next step is to derive the stability conditions to obtain the quadratic forms necessary for a metric. In the process of deriving the equilibrium conditions and in turn the stability conditions other state functions are used. These functions may be defined briefly here as:

Mechanical enthalpy (H): $H \equiv U - Fq$

Mechanical Helmholtz function (K): $K \equiv U - \phi(\dot{q})S$, and

Mechanical Gibbs function (G): $G \equiv H - \phi(\dot{q})S$

These functions may be used to derive Maxwell type relations for a mechanical system and these relations are presented in reference (10) but are not included here.

The derivation of the equilibrium and stability conditions is identical to the derivation of the thermodynamic equilibrium and stability conditions with the variables changed to represent the mechanical variables $\dot{q}$, q, S, and F instead of the thermodynamic variable T, V, S, and P.

1. Equilibrium Conditions

To establish the criteria for equilibrium consider Clausius' theorem

$$\int_{A\,I}^{B} \frac{\bar{d}Q}{\phi} - \int_{A\,R}^{B} \frac{\bar{d}Q}{\phi} \leq 0 ,$$

or

$$\int_{A\,I}^{B} \frac{\bar{d}Q}{\phi} \leq \int_{A\,R}^{B} \frac{\bar{d}Q}{\phi} \equiv S(B) - S(A).$$

For a Q-conservative system $\bar{d}Q = 0$, then

$$\Delta S \geq 0 ,$$

or

$$S(B) \geq S(A).$$

Therefore the mechanical entropy tends toward a maximum so that spontaneous changes in a Q-conservative system will always be in the direction of increasing mechanical entropy.

Now by the first law

$$\Delta Q = \Delta U - F\Delta q.$$

Therefore

$$\phi\Delta S \geq \Delta U - F\Delta q$$

which is analogous to the Clausius inequality in thermodynamics.

Now consider a virtual displacement $(U,q)\rightarrow(U+\delta q,\ q+\delta q)$, which implies a variation $S\rightarrow S+\delta S$ away from equilibrium. The restoration of equilibrium from the varied state $(U+\delta U,\ q+\delta q)\rightarrow(U,q)$ will then certainly be a spontaneous process, and by the Clausius inequality

$$\phi(-\delta S) > -\ (\delta U - F\delta q).$$

Hence, for variations away from equipoise, the general inequality

$$\delta U - F\delta q - \phi\delta S > 0 \tag{6}$$

must hold. The inequality sign is reversed from the sign in Clausius' inequality because hypothetical variations δ away from equilibrium are considered rather than real changes toward equilibrium.

In a spontaneous process,

$$\phi\Delta S \geq \Delta Q_{rev} = \Delta U + \text{work done by the system.}$$

The "work" consists of two parts. One part is the work done by the negative of the force F. It may be positive or negative but it is <u>inevitable</u>. Only the rest is free energy, which is available for some useful work. This latter part may be written as

$$A = \Delta Q_{rev} - \Delta U + F\Delta q.$$

The maximum of A is

$$A_{max} = \phi\Delta S - \Delta U + F\Delta q. \tag{7}$$

which is obtained when the process is conducted reversibly.

The least work, δA_{min}, required for a displacement from equilibrium must be exactly equal to the maximum work in the converse process whereby the system proceeds spontaneously from the "displaced" state to equilibrium (otherwise a perpetual motion machine may be constructed). Corresponding to equation (7) then, is

$$\delta A_{min} = \delta U - F\delta q - \phi\delta S\ .$$

The equilibrium criteria may then be expressed as

$$\delta A_{min} \geq o.$$

In words: At equilibrium the mechanical free energy is a minimum. Any displacement from this state requires work.

2. Stability Conditions

To decide whether or not an equilibrium is stable, the inequality sign in equation (6) must be ensured. The conditions for stability may take different forms depending upon which variables are taken as the independent variables.

To derive the stability conditions when q and S are taken as the independent variables consider the terms of second order in small displacements beginning with the general condition

$$\delta U - F\delta q - \phi\delta S > o .$$

Choose $U = U(q,S)$, which, because of the identity

$$dS = \frac{dU}{\phi} - \frac{F}{\phi}\, dq$$

or

$$\phi dS = dU - Fdq,$$

is a natural choice of the independent variables, and expand δU in powers of δq and δS

$$\delta U = \phi\delta S + F\delta q + \tfrac{1}{2}\left(\frac{\partial^2 U}{\partial q^2}\delta q^2 + 2\frac{\partial^2 U}{\partial q\delta S}\,\delta q\delta S + \frac{\partial^2 U}{\partial S^2}\delta S^2\right) + \text{terms of third order...} \tag{8}$$

The inequality (6) then shows that in (8)

Second order terms + third order terms + ...> 0.

Retaining only the second order terms, the criterion of stability is that a quadratic differential form be positive definite;

$$\frac{\partial^2 U}{\partial q^2}\,\delta q^2 + 2\,\frac{\partial^2 U}{\partial q\partial S}\,\delta q\delta S + \frac{\partial^2 U}{\partial S^2}\,\delta S^2 > 0. \tag{9}$$

If this is to hold true for arbitrary variations in δq and δS, the coefficients must satisfy the following:

$$\frac{\partial^2 U}{\partial q^2} > 0; \ \frac{\partial^2 U}{\partial S^2} > 0; \ \frac{\partial^2 U}{\partial S^2}\frac{\partial^2 U}{\partial q^2} - \left(\frac{\partial^2 U}{\partial q \partial S}\right)^2 > 0.$$

When $\dot{q}$ and q are considered to be the independent variables a quadratic form in $\delta\dot{q}$ and δq may be found by using

$$K = U - \phi S$$

so that

$$\delta K = \delta U - \phi\delta S - \frac{d\phi}{d\dot{q}} S\delta\dot{q} - \frac{d\phi}{d\dot{q}} \delta S\delta\dot{q}.$$

The terms $\delta S\delta\dot{q}$ cannot be neglected because in Clausius' inequality, which is the actual stability condition, the variations are finite, therefore, from equation (6) the following is obtained:

$$\delta K + \phi\delta S + \frac{d\phi}{d\dot{q}}(S + \delta S)\delta\dot{q} - F\delta q - \phi\delta S > 0,$$

$$\delta K + \frac{d\phi}{d\dot{q}} S\delta\dot{q} + \frac{d\phi}{d\dot{q}} \delta S\delta\dot{q} - F\delta q > 0 .$$

Expanding in powers of $\delta\dot{q}$ and δq

$$\delta K = F\delta q - \frac{d\phi}{d\dot{q}}S\delta\dot{q} + \frac{1}{2}\frac{\partial^2 K}{\partial q^2}\delta q^2 + \frac{\partial^2 K}{\partial q\partial\dot{q}}\delta q\delta\dot{q} + \frac{1}{2}\frac{\partial^2 K}{\partial q^2}\delta q^2 + \ldots$$

$$\delta S\delta\dot{q} = \frac{1}{} \frac{\partial U}{\partial\dot{q}}\delta\dot{q}^2 + \frac{1}{\phi}\left(\frac{\partial U}{\partial q} - F\right) \delta q\delta\dot{q}$$

But

$$\frac{\partial K}{\partial\dot{q}} = \frac{\partial U}{\partial\dot{q}} - \frac{\partial\phi}{\partial\dot{q}}S - \frac{\phi}{\phi}\frac{\partial U}{\partial\dot{q}} = -\frac{d\phi}{d\dot{q}}S ;$$

$$\frac{\partial K}{\partial q} = F .$$

Therefore

$$\frac{\partial^2 K}{\partial\dot{q}\partial q} = \frac{\partial F}{\partial\dot{q}} = -\frac{\partial\phi}{\partial\dot{q}}\left(\frac{1}{2}\right)\left(\frac{\partial U}{\partial q} - F\right),$$

and

$$\frac{\partial^2 K}{\partial\dot{q}^2} = -\frac{\partial^2\phi}{\partial\dot{q}^2}S - \frac{d\phi}{d\dot{q}}\left(\frac{1}{\phi}\right)\frac{\partial U}{\partial\dot{q}}$$

then $$\left(\frac{d\phi}{d\dot{q}}\right)\delta S\delta\dot{q} = -\left(\frac{\partial^2\phi}{\partial\dot{q}^2}S + \frac{\partial^2 K}{\partial\dot{q}^2}\right)(\delta\dot{q})^2 - \frac{\partial^2 K}{\partial\dot{q}\partial q}\delta\dot{q}\delta q ,$$

and the quadratic form in $\delta\dot{q}$ and δq is

$$\frac{1}{2}\frac{\partial^2 K}{\partial q^2}(\delta q)^2 + \frac{\partial^2 K}{\partial q\partial\dot{q}}\delta q\delta\dot{q} + \frac{1}{2}\frac{\partial^2 K}{\partial\dot{q}^2}(\delta\dot{q})^2 - \frac{\partial^2 K}{\partial\dot{q}^2}(\delta\dot{q})^2 - \frac{d^2\phi}{d\dot{q}^2}S(\delta\dot{q})^2 - \frac{\partial^2 K}{\partial q\partial\dot{q}}\delta q\delta\dot{q} > 0 ,$$

or

$$\frac{\partial^2 K}{\partial q^2}(\delta q)^2 - \left(\frac{\partial^2 K}{\partial\dot{q}^2} + 2\frac{d^2\phi}{d\dot{q}^2}S\right)(\delta\dot{q})^2 > 0 .$$

Since $\left(\frac{\partial K}{\partial q}\right)_{\dot{q}} = F$ then

$$\frac{\partial^2 K}{\partial q^2} = \left(\frac{\partial F}{\partial q}\right)_{\dot{q}} > 0.$$

Other quadratic forms may be derived by using different independent variables however these two quadratic forms will suffice for this development.

C. GEOMETRY AND FIELD EQUATIONS

If we adopt one of the quadratic forms of the stability conditions as our metric giving the geometry and obtain equations of motion then it becomes obvious that if we choose a form with velocity as one of the coordinates then the equations of motion will become third-order differential equations. The fact that these equations of motion would be third order in time displays the time asymetrical nature that was mentioned in the introduction and taken as a desireable feature of equations with universal application. However the third order nature poses a problem in determining a solution for third order equations can be difficult or impossible to solve.

1. Geometry

For this reason then we shall adopt the quadratic form of equation (9) as the metric for our system. Thus we are adopting a manifold with coordinates of space-mechanical entropy. Caratheodory's proof of the extension of the integrating factor to systems with additional variables may be used to extend the quadratic form to one in four dimensions; three in space plus the entropy. The quadratic form then becomes

$$\frac{\partial^2 U}{\partial S^2}(dS)^2 + \partial \frac{\partial^2 U}{\partial S \partial q}\alpha(dS)(dq^\alpha) + \frac{\partial^2 U}{\partial q^\alpha \partial q^\beta}(dq^\alpha) > 0;\ \alpha,\beta = 1, 2, 3.$$

Adopting this quadratic form as the metric of a general system whose thermodynamic variables are held fixed we may then write this metric as

$$(ds)^2 \equiv h_{ij}\, dq^i\, dq^i;\ (i, j = 0, 1, 2, 3) \tag{10}$$

where the summation convention is used and

$$h_{ij} \equiv \frac{\partial^2 U}{\partial q^i\, \partial q^i}\ ,$$

with $q^0 \equiv S/Fo$, the scaled mechanical entropy for dimensional correctness.

Thus the stability conditions provide a metric in the four-dimensional manifold of space-mechanical entropy. However the existing relativistic theories are theories in a space-time manifold. Therefore if these theories are to be contained within the dynamic theory then the space-time manifold must be found within the dynamic theory.

The arc length s in the space-mechanical entropy manifold may be parameterized by chosing

$$ds \equiv \dot{q}_0\, dt \equiv cdt$$

where $\dot{q}_0 \equiv c$ is the unique velocity appearing in the integrating factor of the second postulate. The metric may now be written as

$$c^2(dt)^2 = h_{ij}\, dq^i dq^j;\ (i, j = 0, 1, 2, 3) \tag{11}$$

Now suppose the systems considered are restricted to only Q-conservative systems. Then the principle of increasing mechanical entropy may be imposed in the form of the variational principle

$$\delta \int \sqrt{(dq^0)^2} = 0.$$

In order to use this variational principle equation (11) may be expanded, solved for (dq^0) and squared to arrive at the quadratic form

$$(dq^o)^2 = (\frac{1}{h_{oo}}) \{c^2(dt)^2 + 2h o\alpha\, A\, dt\, dq^\alpha - h\alpha\beta dq^\alpha dq^\beta\} \tag{12}$$

where

$$A \equiv \frac{h o\gamma \dot{q}^\gamma}{h_{oo}} \pm \sqrt{\frac{c^2}{h_{oo}} \frac{h\gamma\beta \dot{q}^\gamma \dot{q}^\beta}{h_{oo}} + \frac{h o\gamma \dot{q}^\gamma}{h_{oo}}^2}$$

with $\dot{q}^\gamma \equiv dq^\gamma/dt$.

By defining $x^o \equiv ct$, $x^\alpha = q^\alpha$; $\alpha = 1, 2, 3$ then equation (12) may be written as

$$(dq^o)^2 = (\frac{1}{f}) \hat{\hat{g}}_{ij}\, dx^i\, dx^j;\ (i, j = 0, 1, 2, 3) \tag{13}$$

where $f \equiv h_{oo}$. This metric obviously reduces, in the Euclidean limit of constant coefficients, to the metric of Minkowski's space-time manifold of special relativity.

In his General Theory of Relativity, Einstein assumed the space-time manifold to be Riemannian. However this assumption involves the a priori assumption that the scalar product be invariant. This assumption was later questioned by Weyl in his generalization of geometry. From the viewpoint that the adopted postulates should contain the other theories it then becomes desireable to determine whether or not these postulates specify the geometry of the $(dq^o)^2$ space-time manifold.

Recalling equation (13) we can define

$$(dq^o)^2 = (\frac{1}{f}) \hat{\hat{g}}_{ij} dx^i dx^j \equiv (\frac{1}{f}) (d\sigma)^2 \equiv \hat{g}_{ij} dx^i dx^j \tag{14}$$

Now the second postulate guarantees the existence of the function mechanical entropy and that dq^o be a perfect differential, therefore

$$dq^o = q^o{}_i dx^i \tag{15}$$

where $q^o{}_i \equiv \partial q^o/\partial x^i$. Then the exactness of dq^o is stated by

$$q^o_{i/j} - q^o{}_{j/i} = o. \tag{16}$$

By defining the parallel displacement of a vector to be

$$d\xi_i = \hat{\Gamma}^{\ v}_{is}\, dx^s \xi_v \tag{17}$$

and using equations (15) and (16) it may be seen that the connections must be symmetrical, or

$$\hat{\Gamma}^{\nu}_{ik} = \hat{\Gamma}^{\nu}_{ki} \quad . \tag{18}$$

In Weyl's generalization of geometry he found it necessary to assume the symmetry of the connections. He proved a theorem showing that the symmetry of the connections guaranteed the existence of a local Euclidean limiting manifold and used this theorem in support of the symmetry assumption. Here the symmetry is necessitated by the adopted laws.

Suppose now we consider whether order of differentiating the change in entropy makes any difference. Therefore consider the difference

$$\Delta(dq^o)^2 \equiv \frac{\partial^2(dq^o)^2}{\partial x^k \partial x^\ell} - \frac{\partial^2(dq^o)}{\partial x^\ell \partial x^k}$$

Since $(dq^o)^2 = q^o{}_i\, q^o{}_j\, dx^i dx^j$ from equation (15), using equation (14) we find

$$q^o{}_i q^o{}_j = \hat{g}_{ij} \ .$$

Then

$$\frac{\partial(dq^o)^2}{\partial x^k} = (q^o_{j|k}\, q^o_i + q^o_{i|k}\, q^o_j)\, dx^i dx^j + (q^o_k)^2$$

Thus

$$\frac{\partial^2(dq^o)^2}{\partial_x{}^k \partial_x{}^\ell} = [q^o{}_{j|k|\ell}\, q^o_i + q^o_{j|k}\, q^o_{i|\ell} + q^o_{i|k|\ell}\, q^o_j + q^o_{i|k}\, q^o_{j|\ell}]dx^i dx^j$$
$$+ 2q^o_{\ell|k}\, q^o_\ell + 2q^o_{k|\ell}\, q^o_k \ .$$

Likewise

$$\frac{\partial^2(dq^o)^2}{\partial x^\ell \partial x^k} = [q^o_{j|\ell|k}\, q^o_i + q^o_{j|\ell}\, q^o_{i|k} + q^o_{i|\ell\ k}\, q^o_j + q^o_{i|\ell}\, q^o_{j\ k}]\ dx^i dx^j$$
$$+ 2q^o_{k|\ell}\, q^o_k + 2q^o_{\ell|k}\, q^o_\ell \ .$$

Therefore the difference must be

$$\Delta(dq^o)^2 = \{(q^o_{j|k|\ell} - q^o_{j|\ell|k})q^o_i + (q^o_{i|k|\ell} - q^o_{i|\ell|k})q^o_j\}\ dx^i dx^j$$

Using the definition equation (17) we see that

$$dq^o{}_i = \hat{\Gamma}^{r}_{i\delta}\, dx^\delta\, q^o{}_r$$

and $$q^o_{i|k} = q^o_r \hat{\Gamma}^r_{ik}$$

also $$q^o_{k|i} = q^o_r \hat{\Gamma}^r_{ki}.$$

Now
$$q^o_{i|k|\ell} = \frac{\partial}{\partial x^\ell}[q^o_r \hat{\Gamma}^r_{ik}] = q^o_{r|\ell} \hat{\Gamma}^r_{ik} + q^o_r \frac{\partial \hat{\Gamma}^r_{ik}}{\partial x^\ell}$$
$$= q^o{}_\delta \hat{\Gamma}^\delta_{r\ell} \hat{\Gamma}^r_{ik} + q^o_r \frac{\partial \hat{\Gamma}^r_{ik}}{\partial x^\ell}$$
$$= q^o_r \{ \hat{\Gamma}^r_{i\ell} \hat{\Gamma}^\delta_{ik} + \frac{\partial \hat{\Gamma}^r_{ik}}{\partial x^\ell} \}.$$

Similarly
$$q^o_{i|\ell|k} = q^o_r \{\hat{\Gamma}^r_{\delta k} \hat{\Gamma}^\delta_{i\ell} + \frac{\partial \hat{\Gamma}^r_{i\ell}}{\partial x^k} \}.$$

Therefore
$$q^o_j(q^o_{i|k|\ell} - q^o_{i|\ell|k}) = q^o_i\, q^o_r \{ \frac{\partial \hat{\Gamma}^r_{ik}}{\partial x^\ell} - \frac{\partial \hat{\Gamma}^r_{i\ell}}{\partial x^k} + \hat{\Gamma}^r_{\delta\ell} \hat{\Gamma}^\delta_{ik} - \hat{\Gamma}^r_{\delta k} \hat{\Gamma}^\delta_{i\ell} \}.$$

Then defining the vector curvature as
$$\hat{R}^r_{i\ell k} \equiv \frac{\partial \hat{\Gamma}^r_{ik}}{\partial x^\ell} - \frac{\partial \hat{\Gamma}^r_{i\ell}}{\partial x^k} + \hat{\Gamma}^r_{\delta\ell} \hat{\Gamma}^\delta_{ik} - \hat{\Gamma}^r_{\delta k} \hat{\Gamma}^\delta_{i\ell} \tag{19}$$

the difference may be written as
$$\Delta(dq^o)^2 = \{q^o_j\, q^o_r\, \hat{R}^r_{i\ell k} + q^o_i\, q^o_r\, \hat{R}^r_{j\ell k}\}\, dx^i\, dx^j .$$

However, recall that $q^o_i\, q^o_i = \hat{g}_{ij}$ then
$$\Delta(dq^o)^2 = \{\hat{g}_{jr} \hat{R}^r_{i\ell k} + \hat{g}_{ir} \hat{R}^r_{j\ell k}\}\, dx^i dx^j.$$

But $\hat{g}_{ri} = \hat{g}_{ir}$ and $\hat{R}_{ijk\ell} = \hat{g}_{ir} \hat{R}^r_{jk\ell}$

so that $$\Delta(dq^o)^2 = \{\hat{R}_{ji\ell k} + \hat{R}_{ij\ell k}\}\, dx^i dx^j$$

or the difference will vanish if

$$\hat{R}_{ji\ell k} = - \hat{R}_{ij\ell k} .$$

Now since

$$(dq^o)^2 = q_i^o\, q_j^o\, dx^i dx^j = \hat{g}_{ij}\, dx^i dx^j$$

differentiation will result in

$$d(dq^o)^2 = d(q_i^o q_j^o\, dx^i dx^j) = d(\hat{g}_{ij}\, dx^i dx^j)$$

or

$$dq_i^o\, q_j^o\, dx^i dx^j + q_i^o\, dq_j^o\, dx^i dx^j + q_i^o q_j^o\, d(dx^i dx^j)$$
$$= d\hat{g}_{ij}\, dx^i dx^j + \hat{g}_{ij}\, d(dx^i dx^j)$$

which can be written as

$$\hat{\Gamma}^r_{i\delta}\, dx^\delta\, q_r^o q_j^o dx^i dx^j + q_i^o\, \hat{\Gamma}^r_{j\delta}\, dx^\delta q_r^o\, dx^i dx^j + q_i^o q_j^o\, d(dx^i dx^j)$$
$$= d\hat{g}_{ij}\, dx^i dx^j + \hat{g}_{ij}\, d(dx^i dx^j).$$

But

$$\hat{g}_{ij} = q_i^o q_j^o$$

Therefore

$$\hat{\Gamma}^r_{i\delta}\, dx^\delta\, \hat{g}_{rj} + \hat{\Gamma}^r_{j\delta}\, dx^\delta\, \hat{g}_{ir} = d\hat{g}_{ij}$$

or

$$\hat{g}_{rj}\, \hat{\Gamma}^r_{i\delta} + \hat{g}_{ri}\, \hat{\Gamma}^r_{j\delta} = \frac{\partial \hat{g}_{ij}}{\partial x^\delta} \tag{20}$$

$$\hat{\Gamma}_{ji\delta} + \hat{\Gamma}_{ij\delta} = \frac{\partial \hat{g}_{ij}}{\partial x^\delta}$$

Now interchange $ji\delta$ to δij to get

$$\hat{\Gamma}_{\delta ij} + \hat{\Gamma}_{j\delta i} = \frac{\partial \hat{g}_{j\delta}}{\partial x^\delta} \tag{21}$$

then interchange $ji\delta$ to $i\delta j$ so that

$$\hat{\Gamma}_{i\delta j} + \hat{\Gamma}_{\delta ij} = \frac{\partial \hat{g}_{\delta i}}{\partial x^j}. \tag{22}$$

Add equations (21) and (22) and subtract equation (20)

$$\hat{\Gamma}_{i\delta j} + \hat{\Gamma}_{\delta ij} + \hat{\Gamma}_{\delta ij} + \hat{\Gamma}_{j\delta i} - \hat{\Gamma}_{ji\delta} - \hat{\Gamma}_{ij\delta} = \frac{\partial \hat{g}_{\delta i}}{\partial x^j} + \frac{\partial \hat{g}_{j\delta}}{\partial x^i} - \frac{\partial \hat{g}_{ij}}{\partial x^\delta}$$

or $$\hat{\Gamma}_{\delta ij} = \frac{1}{2}\{\frac{\partial \hat{g}_{\delta i}}{\partial x^j} + \frac{\partial \hat{g}_{\delta j}}{\partial x^i} - \frac{\partial \hat{g}_{ij}}{\partial x^\delta}\} \tag{23}$$

and $$\hat{\Gamma}^r_{ij} = \hat{g}^{r\delta}\,\hat{\Gamma}_{\delta ij} = \frac{\hat{g}^{r\delta}}{\partial}\{\frac{\partial \hat{g}_{\delta i}}{\partial x^j} + \frac{\partial \hat{g}_{\delta j}}{\partial x^i} - \frac{\partial \hat{g}_{ij}}{\partial x^\delta}\}.$$

Now by using the symmetrics of $\hat{g}_{ij}$ it can be shown that

$$\hat{R}_{ji\ell k} = -\,\hat{R}_{ij\ell k}$$

and therefore $$\Delta(dq^o)^2 = 0\ .$$

This is the necessary and sufficient condition that the differential entropy change may be transferred from an initial point to all points of the space in a manner that is independent of the path.

The distinguishing features of Riemannian geometry is the invariance of the scalar product under a vector transplantation. Therefore to determine whether the $(dq^o)^2$ space is a Riemannian space consider the vectors $\hat{\xi}_i$ and $\hat{\eta}_i$.

Now since $$\hat{\xi}_i = \hat{g}_{ij}\hat{\xi}^j$$

and $$d\hat{\xi}_i = \hat{\Gamma}^r_{i\delta}dx^\delta\hat{\xi}_r = \hat{\Gamma}^r_{i\delta}dx^\delta\,\hat{g}_{rk}\,\hat{\xi}^k = \frac{\partial \hat{g}_{ij}}{\partial x^\delta}\hat{\xi}^j\,dx^\delta + \hat{g}_{ij}d\hat{\xi}^j$$

then $$\hat{g}_{ij}\,d\hat{\xi}^j = \hat{\Gamma}^r_{i\delta}dx^\delta\,\hat{g}_{rk}\,\hat{\xi}^k - \frac{\partial \hat{g}_{ij}}{\partial x^\delta}\hat{\xi}^j dx^\delta$$

or since $$\hat{g}^{ij}\hat{g}_{ij} = \delta^i_i = 1 \text{ and } \frac{\partial \hat{g}_{ij}}{\partial x^\delta} = \hat{\Gamma}_{ji\delta} + \hat{\Gamma}_{ij\delta}$$

then $$d\hat{\xi}^j = \hat{g}^{ij}\,\{\hat{\Gamma}_{ki\delta}\,\hat{\xi}^k - (\hat{\Gamma}_{ji\delta} + \hat{\Gamma}_{ij\delta})\hat{\xi}^j\}\,dx^\delta$$

$$= \hat{g}^{ij}\,(-\,\hat{\Gamma}_{ik\delta})\,\hat{\xi}^k\,dx^\delta$$

$$= -\,\hat{\Gamma}^j_{k\delta}\,dx^\delta\,\hat{\xi}^k\ .$$

Thus the change in the covariant and the contravariant vectors is given by

$$d\hat{\xi}_i = \hat{\Gamma}^r_{i\delta}\, dx^\delta\, \hat{\xi}_r$$

and

$$d\hat{\xi}^i = -\hat{\Gamma}^i_{r\delta}\, dx^\delta \hat{\xi}^r \quad .$$

Now consider the change in the scalar product $\hat{\xi}_i\, \hat{\eta}^i$. Then

$$d(\hat{\xi}_i\hat{\eta}^i) = d\hat{\xi}_i\hat{\eta}^i + \hat{\xi}_i d\hat{\eta}^i$$

$$= \hat{\Gamma}^r_{i\delta}\, dx^\delta\, \hat{\xi}_r\hat{\eta}^i + \hat{\xi}_i\, (-\hat{\Gamma}^i_{r\delta} dx^\delta\, \hat{\eta}^r)$$

$$d(\hat{\xi}_i\hat{\eta}^i) = \hat{\Gamma}^r_{i\delta} dx^\delta \hat{\xi}_r\hat{\eta}^i - \hat{\Gamma}^i_{r\delta}\, dx^\delta \hat{\xi}_i\hat{\eta}^r.$$

Renaming the indices in the second term yields

$$d(\hat{\xi}_i\hat{\eta}^i) = (\hat{\Gamma}^r_{i\delta}\, \hat{\xi}_r\hat{\eta}^i - \hat{\Gamma}^r_{i\delta}\, \hat{\xi}_r\hat{\eta}^i)dx^\delta$$

$$\equiv 0.$$

Thus the geometry of the $(dq^0)^2$ manifold is Riemannian.

The next question is what is the geometry of the $(d\sigma)^2$ space? Using equation (14) $(d\sigma)^2$ may be written as $(d\sigma)^2 = f(dq^0)^2$. The appearance of the gauge function f means that by forming the difference $\Delta(d\sigma)^2$ then it may be shown that the vanishing of $f_{k/\ell} - f_{\ell/k}$ is the necessary and sufficient condition that every differential distance $d\sigma$ may be transferred from an initial position to all points in the $(d\sigma)^2$ manifold in a manner independent of the path.

Defining the potential $\phi_k \equiv \pm \frac{\partial \ell n f^{\frac{1}{2}}}{\partial x^k}$ the change of $(d\sigma)$ may be written as

$$d(d\sigma) = \phi_k\, dx^k\, (d\sigma). \tag{24}$$

If $\phi_{k/j} - \phi_{j/k}$ vanishes then the $(d\sigma)^2$ manifold becomes a Weyl space. Using the connections $\hat{\hat{\Gamma}}{}^{v}_{k\ell}$ in the $(d\sigma)^2$ manifold we then have

$$\phi_{k/j} - \phi_{j/k} = (\hat{\hat{\Gamma}}{}^v_{kj} - \hat{\hat{\Gamma}}{}^v_{jk})\phi_v.$$

However by considering $d(d\phi)^2 = d(\hat{\hat{g}}_{ij}dx^i dx^j)$ the expressions for the connections $\hat{\hat{\Gamma}}^i_{jk}$ may be shown to be Weyl's connections given by

$$\hat{\hat{\Gamma}}^i_{jk} = \hat{\Gamma}^i_{jk} + \frac{\hat{\hat{g}}^{is}}{2} \{ \hat{\hat{g}}_{sj}\, \phi_k + \hat{\hat{g}}_{sk}\, \phi_j - \hat{\hat{g}}_{jk}\, \phi_s \} \quad . \tag{25}$$

Now since $\hat{\Gamma}^i_{jk} = \hat{\Gamma}^i_{kj}$ and the symmetry of the second term on the right hand side is obvious then

$$\hat{\hat{\Gamma}}^i_{jk} = \hat{\hat{\Gamma}}^i_{kj} .$$

Thus $\phi_{k/\ell} - \phi_{\ell/k} \equiv o$ and the $(d\sigma)^2$ manifold must be a Weyl geometry with the quadratic form $(d\sigma)^2$ and the linear differential form $d\phi = \phi_k dx^k$.

2. Field Equations

We have now shown that the entropy space is a Riemannian space while the sigma space is a Weyl space. There remains the question of equations of motion and field equations. The answer to these questions is provided by Weyl's unified field theory for the geometrical principles we have derived plus the variational principle provided by the principle of increasing mechanical entropy provide the set of postualtes Weyl made in his theory. We therefore arrive at the variational problem

$$\delta \int [\hat{R} + \tfrac{1}{2} A\, \tilde{F}_{ij}\tilde{F}^{ij} - \lambda(\tfrac{1}{2} - 6\, \tilde{\phi}_i\tilde{\phi}^i)]\, \sqrt{-\hat{g}}\; d^4 x = 0 \tag{26}$$

where $\phi_i \equiv \sqrt{\lambda}\, \tilde{\phi}_i$ and $F_{ij} \equiv \sqrt{\lambda}\; \tilde{F}_{ij}$ and the $\tilde{\phi}_i$ and $\tilde{F}_{ij}$ are the components of the vector potential and the electromagnetic field measured in the usual units.

By varying only the potentials in equation (26) we obtain Maxwell's electromagnetic theory. Variation of the $\hat{g}_{ij}$ produces Einstein's General Relativisitc theory.

Thus the Dynamic Theory yields field equations of Maxwell's electromagnetic theory and Einstein's general relativistic theory. There is, however, an additional benefit achieved here. Recall that in both the entropy and sigma manifolds we showed that the order of differentiation of the element of arc length was immaterial. This is important for the following reason. Einstein objected to Weyl's unified theory based upon an argument that using Weyl's theory one should expect the spectral lines produced from an atom changing states would depend upon the histories of the atom and we should not see the sharp lines we observe. From this theories point of view the order of differentiation becomes important for the change of the entropy of the atom produces the spectral lines and we have shown that not only the entropy but the change in the entropy is independent of the path. Therefore the spectral lines would be expected to be independent of the atom's history and hence the sharp lines we observe.

D. QUANTUM EFFECTS

In 1927 F. London derived quantum principles from Weyl's theory. However London's result made it difficult to define length as a real number. Because of this Weyl interpreted the mathematical formulism of his unified theory as connected with transplanting a state vector of a quantum-theoretical system.

The Dynamic Theory removes the difficulty of defining real lengths from London's results. This may be demonstrated by considering a Q-conservative, or isolated, system. For this system, since $\bar{d}Q = o$, the second postulate requires

$$dq^o \geq 0$$

which is the principle of increasing mechanical entropy. Then certainly $(dq^o)^2 \geq 0$, and also, since

$$(dq^o)^2 = f(d\sigma)^2 ,$$

$$f(d\sigma)^2 \geq o .$$

However if $f < o$ then $(d\sigma)^2 < o$ since it is the product which must remain greater than or equal to zero. In this case

$$dq^{o} = \sqrt{-f} \quad \sqrt{-(d\sigma)^2} \ .$$

But by integrating equation (16) we find that the element of arc $(d\sigma)$ is given by

$$(d\sigma) = (d\sigma)_o \, e^{\int \phi_k dx^k}$$

where $(d\sigma)_o$ is some initial value of the element of arc. If $(d\sigma) = (d\sigma)_o$ then the integral in the exponent must be equal to $2\pi i n$ which is the quantization London introduced.

To illustrate how this condition arises naturally from the dynamic approach suppose a description of a hydrogen atom is desired. A hydrogen atom is in a stable condition and if isolated satisfies the conditions $\bar{d}Q = o$ and $dq^{o} = o$ since the principle of increasing entropy requires that the entropy be a maximum at equilibrium for an isolated system. These conditions along with $f \neq o$ establish the quantization of the integral $\int \phi_k dx^k$. If the field of the proton is taken as $\phi_o = \alpha'/r$; $\phi_\alpha \equiv o$; $\alpha \neq o$ then simple circular motion produces Bohr radii for $\alpha' = \frac{2\pi i e^2}{hc}$ where h is Planck's constant. The imaginary α' presented the difficulty of defining length as a real number. In the dynamic approach real distance, or length, may, and properly should, be defined in the $(dq^{o})^2$ manifold. Recalling the definition of the potentials it may easily be seen that if $f < o$ then the arc length given by

$$\sigma = \int \sqrt{(d\sigma)^2}$$

will be imaginary. However the arc length in the $(dq^{o})^2$ manifold is real since $dq^{o} \geq o$.

E. SUMMARY

The First Law adopted here is a statement of the notion of conservation of energy and as such contains nothing essentially new. It is important however to note that the Q-conservative system, where $\bar{d}Q = o$, does not necessarily correspond to the usual notion of a "conservative" system in classical mechanics. In classical mechanics a "conservative" system is one for which the system's energy does not change. Therefore a system within the Dynamic Theory which corresponds to the classical notion of a "conservative" system is one for which $dU \equiv 0$. From our experience with thermodynamic systems we can see that this is somewhat different from a Q-conservative system.

The Second Law enables us to find an integrating factor for the First Law and in so doing answers the question posed in the introduction concerning the speed of light as a limiting velocity for forces other than electromagnetic. It was shown that the integrating factor was independent of the force and therefore does not depend upon what type of force is considered. The absolute velocity is defined as that constant velocity process for which the integrating factor is zero. Hence the absolute velocity is independent of the force and therefore must be a unique velocity applicable to all forces.

Since by definition the absolute velocity is a constant in one reference frame it must also be a constant in any other reference frame moving with a constant velocity relative to the first. Thus the absolute velocity must be unique and a constant in all references frames moving with constant relative velocities. The experimental and theoretical evidence of electromagnetism requires that the speed of light obey these same properties. Thus the absolute velocity must be the speed of light and act as a limiting velocity for all forces.

The process, or procedure, of using the quadratic form as the metric is not known (by this author) to have been used before. However its use, through the geometry and principle of increasing entropy, leads us rather naturally to the relativistic theories, electromagnetism, and quantum effects.

Three points here seem particularly significant. First, not only does the adopted laws yield Weyl's unified theory but they completely specify the geometrical assumptions of Weyl's. This removes the necessity of making any assumptions concerning the geometry. Thus Einstein's assumption of a Riemannian space applies only to the entropy manifold for a Q-conservative system, but is necessary for that manifold. The second point is that while there may be several sets of constraints which require that $(d\sigma) = (d\sigma)_0$ which produces quantum effects these effects apply only to forces which may be described in terms of Weyl's "distance curvature" and not to forces describable by a "vector curvature". Or, in terms of the interpretations of Weyl's unified theory, quantum effects may be seen for electromagnetic forces but not for gravitational thus providing an explanation for the resistance gravitational effects have put up against quantization. Another aspect of this point is its support for Einstein's celebrated quote, "God doesn't play dice." According to the Dynamic Theory quantum effects is required but only for systems subjected to certain restrictions. The more general system however is not subjected to quantization. Thus while the Dynamic Theory supports, and indeed requires, quantum effects it also supports Einstein's contention that everything should not be quantized.

The third point is that the adopted laws can produce the different branches of physics currently used. In unifying the different branches of physics in this manner the Dynamic Theory may prove beneficial not only in better understanding the inter-relationships of the various branches but allows the possibility of using well developed techniques of one branch, such as equations of motion in mechanics, in another branch, such as non-equilibrium thermodynamics.

III. FIVE-DIMENSIONAL SYSTEMS

A. SYSTEMS NEAR AN EQUILIBRIUM STATE

Having established the unifying aspect of the Dynamic Theory it seems reasonable to make the next step that of considering a system with both thermodynamic and mechanical variables in order to display the effects of applying the Dynamic Theory to a more general system. This extension will be done as a series of three steps because different restrictions on the system has the apparent effect of extending existing theories. This manner of presentation should allow readers specializing in different fields of physics to quickly see the effect of the Dynamic Theory in their area of specialization.

1. Equations of Motion

In Chapter II, it was shown how the set of three laws generalized from the classical laws may be used to specify the geometry of the system and how these laws require the existing theories of classical thermodynamics, Special and General relativity, Maxwell's electromagnetism and quantum effects. During this development strict adherence to a separation of thermodynamic and mechanical variables was maintained. In reality this separation can hardly be considered as total or complete, for instance in a plasma subjected to electromagnetic fields the density and temperature may vary considerably.

Therefore suppose we consider some sort of plasma, which may contain a charge density, in order to see what the established procedure of the Dynamic Theory may yield. Some of the benefits which may be expected to result from such an investigation might be a better description of reality by logically connecting the thermodynamic and mechanical variables, standard procedures and equations for non-equilibrium thermodynamic systems, and a new insight into the electromagnetic containment of a plasma.

For a system with thermodynamic as well as mechanical variables the first law becomes

$$\bar{d}\tilde{Q} = d\tilde{U} + Pdv = \tilde{F}_\alpha \, dq^\alpha; \ \alpha = 1, 2, 3.$$

Where the $\tilde{Q}$, $\tilde{U}$, v and $\tilde{F}_\alpha$ are considered as specific quantities. That is these quantities are related to a unit of mass such as is customary in thermodynamics.

The specific volume is the reciprocal of the mass density, γ, then using the mass density instead of the specific volume the first law becomes,

$$\bar{d}\tilde{Q} = d\tilde{U} - (P/\gamma^2)d\gamma - \tilde{F}_\alpha \, dq^\alpha; \ \alpha = 1, 2, 3.$$

This law now requires that the system's energy $\tilde{U}$ be a function of five independent variables so that

$$\tilde{U} = \tilde{U}\ (\tilde{S}, q^1, q^2, q^3, \gamma).$$

Thus the first law requires a five-dimensional manifold of entropy, space, and mass for a general system. Since the system under consideration needs both thermodynamic and mechanical variables we can no longer refer to the entropy as mechanical or thermodynamic however, the limiting case where the mass is held fixed must produce the mechanical entropy.

The procedure established by the Dynamic Theory is to take the stability condition quadratic form as the metric for a stable system. Thus the coefficients of the metric become the second partial derivatives of the energy function. In order to simplify the metric suppose for the present that we restrict our system to be very near an equilibrium state so that we may consider the second partial derivatives to be constants. This is in essence considering a local Euclidean manifold which the symmetry of the geometric connections guarantees that we may do.

Since the metric coefficients are constants a transformation may be found such that the cross terms are zero. Then in this coordinate system the metric becomes

$$c^2(dt)^2 = (dq^0)^2 + dq^\alpha dq^\alpha + (dq^4)^2; \ \alpha = 1,\ 2,\ 3. \tag{27}$$

when

$$q^0 \equiv \frac{\tilde{S}}{F_0} \quad \text{and} \quad q^4 \equiv \frac{\Upsilon}{a_0}$$

If we again consider the restriction $\bar{d}\tilde{Q} = 0$ so that we are talking of a Q-conservative system for which the principle of increasing entropy holds, then we have the variational principle given by

$$\delta \int \sqrt{(d\tilde{S})^2} = 0. \tag{28}$$

Solving equations (27) for dq^0 and squaring we get

$$(dq^0)^2 = c^2(dt)^2 - dq^\alpha dq^\alpha - (dq^4)^2 \tag{29}$$

or

$$\left(\frac{dq^0}{dt}\right)^2 = c^2 - g_{\alpha\beta} \left(\frac{dq^\alpha}{dt}\right) \left(\frac{dq^\beta}{dt}\right); \ \alpha,\ \beta = 1,\ 2,\ 3,\ 4$$

$$g_{\alpha\beta} = \delta_{\alpha\beta}.$$

The entropy manifold given by equation (29) is a five-dimensional Minkowski-type manifold with coordinates of space-time-mass. We may therefore follow the procedure Minkowski and Einstein used in the Special Theory of Relativity.

First, to avoid confusion, let us rename the coordinates as

$$x^0 \equiv ct;\ x^1 \equiv q^1,\ x^2 \equiv q^2,\ x^3 \equiv q^3 \text{ and } x^4 \equiv q^4.$$

Then define the five-dimensional velocity vector as

$$u^i \equiv \frac{dx^i}{dq^0}; \ i = 0,\ 1,\ 2,\ 3,\ 4$$

and define the five-dimensional acceleration vector as

$$f^i \equiv \frac{\delta u^i}{\delta q^o} \equiv \frac{d^2x^i}{dq^{o2}} + \{{}^{\,i}_{\ell k}\} \frac{dx^\ell}{dq^o} \frac{dx^k}{dq^o}.$$

Now the specific entropy is the arc length and the variational principle is based upon the entropy. Therefore if we multiply the specific entropy by the mass density we have the entropy density. The variational problem becomes

$$\delta \int \sqrt{\gamma^2(dq^o)^2} = \delta \int \gamma \sqrt{(dq^o)^2} = 0. \tag{30}$$

The Euler equations for this problem are

$$\frac{d}{dq^o} \left\{\frac{\gamma\, g_{ij}u^j}{\sqrt{g_{ij}u^iu^j}}\right\} - \frac{\partial\gamma}{\partial x^i} \sqrt{g_{ij}u^iu^j} - \frac{\gamma \dfrac{\partial g_{ik}}{\partial x^i} u^iu^k}{\sqrt{g_{ij}u^iu^j}} = 0$$

or

$$a_o\, u^4 \frac{g_{ij}u_j}{\sqrt{g_{ij}u^iu^j}} - \frac{\partial\gamma}{\partial x^i} \sqrt{g_{ij}u^iu^j} + \gamma \left\{\frac{d}{dq^o} \left[\frac{g_{ij}u^j}{\sqrt{g_{ij}u^iu^j}}\right] - \frac{\dfrac{\partial g_{ik}}{\partial x^i} u^iu^k}{\sqrt{g_{ij}u^iu^j}}\right\} = 0.$$

Using the fact that $g_{ij}u^iu^j = 1$ the Euler equations become

$$\gamma f^i = \frac{\partial\gamma}{\partial x^i} - a_o\, u^4\, g_{ij}u^j \equiv F^i \tag{31}$$

where the F^i are force densities.

Obviously if we hold the mass density fixed, $u^4 \equiv 0$, then the volume integral of this equation becomes the force-mass-acceleration relationship of special relativity.

Now since $f^i = \frac{\delta u^i}{\delta q^o}$ and $(\frac{dq^o}{dt}) = c^2 - u^\alpha u^\alpha$; $\alpha = 1, 2, 3, 4,$

then

$$F^i = \gamma \frac{\delta u^i}{\delta q^0} = \gamma \frac{\delta u^i}{\delta t} \frac{dt}{dq^0}$$

$$= \frac{\gamma}{\sqrt{c^2 - v^2}} \frac{\delta}{\delta t} \left(\frac{dx^i}{dq^0}\right) \text{ where } v^2 = u^\alpha u^\alpha;\ \alpha = 1, 2, 3, 4.$$

Then

$$F^\alpha = \frac{\gamma}{\sqrt{c^2 - v^2}} \frac{\delta}{\delta t} \left(\frac{1}{\sqrt{c^2 - v^2}} \frac{dx^\alpha}{dt}\right)$$

$$= c^2 \frac{\gamma}{\sqrt{1 - \beta^2}} \frac{\delta}{\delta t} \left(\frac{1}{\sqrt{1 - \beta^2}} \frac{dx^\alpha}{dt}\right)$$

where $\beta \equiv v/c$ with v the four-dimensional speed.

The force density equation may now be written as

$$\sqrt{1 - \beta^2}\ F^\alpha = \frac{\gamma}{c^2} \frac{\delta}{\delta t} \left(\frac{1}{\sqrt{1 - \beta^2}} \frac{dx^\alpha}{dt}\right).$$

Consider

$$\frac{\delta}{\delta t} \left[\frac{\gamma}{\sqrt{1 - \beta^2}} \frac{dx^\alpha}{dt}\right] = \frac{\delta}{\delta t} (\gamma) \left[\frac{1}{\sqrt{1 - \beta^2}} \frac{dx^\alpha}{dt}\right] + \frac{\gamma\delta}{\delta t} \left[\frac{1}{\sqrt{1 - \beta^2}} \frac{dx^\alpha}{dt}\right]$$

but $\frac{\delta(\gamma)}{\delta t} = a_o v^4$ so that the force density equations may now be written as

$$\sqrt{1 - \beta^2}\ F^\alpha = \frac{1}{c^2} \frac{\delta}{\delta t} \left[\frac{\gamma}{\sqrt{1 - \beta^2}} \frac{dx^\alpha}{dt}\right] - \frac{a_o u^4}{c^2} \left[\frac{1}{\sqrt{1 - \beta^2}} \frac{dx^\alpha}{dt}\right].$$

We may define $\frac{\gamma}{\sqrt{1 - \beta^2}} \equiv \gamma^1$ as the effective mass density or "Relativistic" mass density then

$$\sqrt{1 - \beta^2}\ F^\alpha = \frac{1}{c^2} \frac{\delta}{\delta t} \left[\gamma^1 \frac{dx^\alpha}{dt}\right] - \frac{a_o v^4 v^\alpha}{c^2 \sqrt{1 - \beta^2}}$$

By defining $F^\alpha \equiv c^2 \sqrt{1 - \beta^2}\ (F^\alpha)$ so that

$$F^{\alpha} = \frac{\delta}{\delta t} [\gamma^{1} \frac{dx^{\alpha}}{dt}] - \frac{a_o\ v^4 v^{\alpha}}{\sqrt{1 - \beta^2}}. \qquad (32)$$

We see that this force density becomes Einstein's special relativistic force density when $v^4 = 0$, or for constant "rest mass." Thus the equations of motion, equation (32) reduce to Einstein's special relativistic equations of motions when $\dot{\gamma} \equiv 0$.

2. Energy Equation

Now for our system the restriction that

$$\bar{d}\tilde{Q} = 0 = d\tilde{U} - \frac{p}{\gamma^2} d\gamma - \tilde{F}_{\alpha}\ dx^{\alpha}: \quad \alpha = 1, 2, 3$$

requires that

$$d\tilde{U} = \frac{p}{\gamma^2} d\gamma + \tilde{F}_{\alpha} dx^{\alpha}; \ \alpha = 1, 2, 3$$

or if $\frac{p}{\gamma^2}$ is considered as another generalized force density then

$$d\tilde{U} = \tilde{F}_{\alpha} dx^{\alpha}; \ \alpha = 1, 2, 3, 4.$$

Thus by integrating the expression for the system's energy change we should arrive at the Einstein energy equation if we hold $\mathring{u}^4 \equiv 0$. Therefore we shall perform the integration using the force densities given by equation (32) to get the system's energy, or

$$\tilde{U} - \tilde{U}_o = \int_{p_o}^{p} \tilde{F}_{\alpha} dx^{\alpha} = \int_{p_o}^{p} \{\frac{d}{dt} [\frac{\gamma}{\sqrt{1 - \beta^2}} \frac{dx^{\alpha}}{dt}] - \frac{a_o\ u^4 u^{\alpha}}{\sqrt{1 - \beta^2}}\}\ dx^{\alpha}$$

$$= \int_{p_o}^{p} \{\frac{d}{dt} \gamma \frac{1}{\sqrt{1 - \beta^2}} \frac{dx^{\alpha}}{dt} + \gamma \frac{d}{dt} [\frac{1}{\sqrt{1 - \beta^2}} \frac{dx^{\alpha}}{dt}] - \frac{a_o\ u^4 u^{\alpha}}{\sqrt{1 - \beta^2}}\}\ dx^{\alpha}$$

$$= \int_{t_o}^{t} \{\gamma \frac{d}{dt} [\frac{1}{\sqrt{1 - \beta^2}} \frac{dx^{\alpha}}{dt}] \frac{dx^{\alpha}}{dt}\}\ dt$$

$$= \int_{t_o}^{t} \{\gamma [\frac{d}{dt} (\frac{1}{\sqrt{1 - \beta^2}})\ u^{\alpha} u^{\alpha} + \frac{u^{\alpha} \frac{du^{\alpha}}{dt}}{\sqrt{1 - \beta^2}}]\}\ dt.$$

But $c^2\beta^2 = u^\alpha u^\alpha$ and $c^2\beta\dot{\beta} = u^\alpha \frac{du^\alpha}{dt}$, therefore

$$\tilde{u} - \tilde{u}_o = \int_{t_o}^{t} \{\gamma\ [\frac{d}{dt}\ (\frac{1}{\sqrt{1 - \beta^2}})\ c^2\beta^2 + \frac{c^2\beta\dot{\beta}}{1 - \beta^2}]\}\ dt$$

$$= c^2 \int_{t_o}^{t} \gamma\ \frac{\beta d\beta}{(1 - \beta^2)^{3/2}}.$$

Now β depends upon u^α and not upon x^4, or γ, therefore

$$\tilde{u} - \tilde{u}_o = \frac{\gamma c^2}{(1 - \beta^2)^{1/2}}$$

or

$$\tilde{u} = \frac{\gamma c^2}{(1 - \beta^2)^{1/2}} + \text{constant}.$$

If the internal energy is considered as the system's energy when the spacial velocities u^α; $\alpha = 1, 2, 3$ are taken as zero then the internal energy density given by

$$\tilde{u} = \frac{\gamma c^2}{\sqrt{1 - (\frac{u^4}{c})^2}} + \text{constant}.$$

At the equilibrium condition where u^4 is also zero the internal energy density is then

$$\tilde{u} = \gamma c^2 + \text{constant}.$$

By taking the constant of integration to be zero this internal energy density then corresponds to Einstein's "rest energy" where here the 'rest energy' is in terms of a four-dimensional "at rest" state.

If we make the usual type approximation of allowing $\beta^2 << 1$ then the system's energy density is approximately given by

$$\tilde{u} = \gamma c^2 + \frac{1}{2}\ \gamma\ v^2 + \frac{1}{2}\ \frac{\gamma}{(a_o)^2}\ (\dot{\Upsilon})^2$$

where here $u^4 \equiv \frac{\dot{\Upsilon}}{a_o}$ is used. This displays the classical limit system energy density for a Q-conservative system very near equilibrium.

The question may be asked whether or not it is necessary to have five dimensions. In considering this question suppose we make the usual definition of temperature as

$$\frac{\partial \tilde{U}}{\partial \tilde{S}} \equiv T = \frac{d\tilde{U}}{dt}\frac{dt}{d\tilde{S}}.$$

Now

$$\frac{d\tilde{U}}{dt} = \frac{\dot{\gamma} c^2}{\sqrt{1 - \beta^2}} - \frac{\gamma\beta\dot{\beta}c^2}{(1 - \beta^2)^{3/2}}$$

and

$$\frac{d\tilde{S}}{dt} = \frac{1}{F_o}\frac{dq^0}{dt} = \frac{c}{F_o}\sqrt{1 - \beta^2}\,.$$

Therefore

$$\frac{dt}{d\tilde{S}} = \frac{F_o}{c\sqrt{1 - \beta^2}}$$

and the temperature becomes

$$T = \left[\frac{\dot{\gamma} c^2}{\sqrt{1 - \beta^2}} - \frac{\gamma\beta\dot{\beta}}{(1 - \beta^2)^{3/2}}\right]\frac{F_o}{c(1 - \beta^2)^{1/2}}$$

$$= \frac{F_o}{c(1 - \beta^2)^2}\left[\dot{\gamma} c^2\,(1 - \beta^2) - \gamma\beta\dot{\beta}\right].$$

For $\beta << 1$ then

$$T \cong \frac{F_o}{c}\left[\dot{\gamma} c^2 - \frac{\gamma\dot{\gamma}\ddot{\gamma}}{(a_o)^2}\right] = \frac{F_o\dot{\gamma}}{c}\left[c^2 - \frac{\ddot{\gamma}}{(a_o)^2}\right].$$

Then as the time rate of change in the mass density approaches zero at the equilibrium state we find that the temperature approaches zero. Physically we do not see this happening therefore a total separation of thermodynamic and mechanical variables is not strictly possible. This requires that we must make an extension of our world from the four dimensions of relativity to the five dimensions of the Dynamic Theory.

B. SYSTEMS WITH NON-EUCLIDEAN MANIFOLD

1. General Variational Principle

Suppose now we relax the assumption that the system is very near an equilibrium point so that the second partial derivatives are no longer constants but are functions. This is essentially the same transition as Einstein made going from his special to general theory, however, the logic of the transition is much simplier here. The only change in the logic appears in the relaxation of the assumption of nearness. There is, of course, a drastic increase in mathematical difficulty since the metric components are no longer constants.

We shall consider a system, which again may be a charged plasma, which must be described by both thermodynamic and mechanical variables. When written in terms of the mass density the first law for this system may be written as

$$d\tilde{Q} = d\tilde{U} - \frac{P}{\gamma^2}\, d\gamma - \tilde{F}_\alpha dq^\alpha;\ \alpha = 1, 2, 3$$

where the tilda denotes specific quantities.

Following the prescribed procedures of the Dynamic Theory we shall take the stability condition quadratic form as the metric for our system. Thus the metric coefficients will be given by the second partial derivatives

$$h_{ij} = \frac{\partial^2 U^2}{\partial q^i \partial q^j};\ i, j = 0, 1, 2, 3, 4$$

where $q^4 \equiv \frac{\gamma}{a_o}$. The metric may then be written as

$$c^2\,(dt)^2 = h_{oo}\,(dq^o)^2 + 2\,h_{o\alpha}\,dq^o dq^\alpha + h_{\alpha\beta}\,dq^\alpha dq^\beta$$

where $\alpha, \beta = 1, 2, 3, 4$.

Imposing the restriction that the system be Q-conservative, $\bar{d}Q = 0$, results in the principle of increasing entropy so that

$$\delta \int \sqrt{(dS)^2} = 0.$$

Thus in terms of the specific entropy the variational principle may be written as

$$\delta \int \sqrt{(\gamma dq^0)^2} \quad = \quad \delta \int \gamma \sqrt{(dq^0)^2} = 0$$

Solving the metric given by equation (30) and squaring yields the expression

$$(dq^0)^2 = \{\frac{1}{h_{00}}\} \{c^2 (dt)^2 + 2 h_{0\alpha} [*] dt dq^\alpha - h_{\alpha\beta} dq^\alpha dq^\beta\}; \alpha, \beta = 1, 2, 3, 4$$

with

$$[*] \equiv \frac{h_{0\gamma}}{h_{00}} q^\gamma \pm \sqrt{\frac{c^2}{h_{00}} - \frac{h_{\gamma\delta}}{h_{00}} \dot{q}^\gamma \dot{q}^\delta + \frac{(h_{0\gamma}\dot{q}^\gamma)^2}{(h_{00})^2}}.$$

This metric in a five-dimensional manifold of space-time-mass may be rewritten as

$$(dq^0)^2 = (\frac{1}{h_{00}}) (d\sigma)^2$$

where

$$(dq^0)^2 \equiv \hat{q}_{ij} dx^i dx^j; i, j = 0, 1, 2, 3, 4$$

$$(d\sigma)^2 \equiv \hat{\hat{q}}_{ij} dx^i dx^j; i, j = 0, 1, 2, 3, 4$$

with $x^0 \equiv ct$; $x^1 \equiv q^1$; $x^2 \equiv q^2$; $x^3 \equiv q^3$; $x^4 \equiv \gamma/a_0$. Thus we may write

$$(dq^0)^2 = \hat{q}_{ij} dx^i dx^j = (\frac{1}{f}) (d\sigma)^2 = (\frac{1}{f}) \hat{\hat{g}}_{ij} dx^i dx^j. \qquad (33)$$

Having established the metrics in equation (33) in the manner prescribed by the Dynamic Theory the geometry must be Weyl geometry and defining the potential five-vector as

$$\phi_i \equiv \pm \frac{\partial \ln f^{1/2}}{\partial x^i} \qquad (34)$$

and the field tensor as

$$F_{ij} \equiv \phi_{i,j} - \phi_{j,i} \tag{35}$$

then we may follow Weyl's procedure in his unified field theory[2] to arrive at the variational principle

$$\delta \int [\hat{R} + \frac{1}{2} A \tilde{F}_{ij}\tilde{F}^{ij} - \lambda(\frac{1}{2} - 12 \tilde{\phi}_i\tilde{\phi}^i)] \sqrt{-\hat{\hat{g}}}\, dx^5 = 0 \tag{36}$$

where $\tilde{F}_{ij} \equiv \frac{1}{\sqrt{\lambda}} F_{ij}$ and $\tilde{\phi}_i \equiv \frac{1}{\sqrt{\lambda}} \phi_i$.

Varying the metric coefficients $\hat{\hat{g}}_{ij}$ in the variational principle (36) will yield field equations of the Dynamic Theory which are extensions of Einstein's General Theory of Relativity.

2. Gauge Function Field Equations

In order to isolate the field equations resulting from a gauge function from the field equations produced by a vector curvature let us consider a Local Euclidean manifold for $(d\sigma)^2$.

Now the field tensor given by equation (35) has 25 components. We would like to determine the field equations for these components. The quickest, though not the only, way is to consider the five dimensions to be

$$x^0 = i\, c\, t; \; x^\alpha = x^\alpha; \; \alpha = 1, 2, 3, 4.$$

The field tensor is then defined to be

$$F_{ij} \equiv \begin{pmatrix} 0 & i\,E_1 & i\,E_2 & i\,E_3 & i\,V_0 \\ -i\,E_1 & 0 & B_3 & -B_2 & V_1 \\ -i\,E_2 & -B_3 & 0 & B_1 & V_2 \\ -i\,E_3 & B_2 & -B_1 & 0 & V_3 \\ -i\,V_0 & -V_1 & -V_2 & -V_3 & 0 \end{pmatrix}.$$

Using Bianchi's identities

$$\frac{\partial F_{ij}}{\partial x^k} + \frac{\partial F_{jk}}{\partial x^i} + \frac{\partial F_{ki}}{\partial x^j} = 0$$

and the various combinations of the indices 0, 1, 2, 3, 4 we obtain the field equations

$$\bar{\nabla} \cdot \bar{B} = 0 \qquad \bar{\nabla} \times \bar{E} + \frac{1}{c}\frac{\partial \bar{B}}{\partial t} = 0 \tag{37}$$

$$\bar{\nabla} \times \bar{V} + a_0 \frac{\partial \bar{B}}{\partial \gamma} = 0 \qquad \bar{\nabla} V_0 + \frac{1}{c}\frac{\partial \bar{V}}{\partial t} + a_0 \frac{\partial \bar{E}}{\partial \gamma} = 0.$$

The definition of the 5-vector current density

$$\frac{\partial F_{ij}}{\partial x^i} \equiv \frac{4\pi}{c} J_i \tag{38}$$

yields the equations

$$\bar{\nabla} \cdot \bar{E} + a_0 \frac{\partial V_0}{\partial \gamma} = 4\pi\rho \qquad \bar{\nabla} \times \bar{B} - \frac{1}{c}\frac{\partial \bar{E}}{\partial t} + a_0 \frac{\partial \bar{V}}{\partial \gamma} = \frac{4\pi \bar{J}}{c} \tag{39}$$

$$\bar{\nabla} \cdot \bar{V} + \frac{1}{c}\frac{\partial V_0}{\partial t} = -\frac{4\pi}{c} J_4.$$

Equations (37) and (39) form a set of seven Maxwell-type equations which obviously reduce to Maxwell's four equations if the mass density is held fixed.

The wave equations for the new field quantities may be derived using standard assumptions.

$$\frac{\partial}{\partial t}(\bar{\nabla} \cdot \bar{V}) + \frac{1}{c}\frac{\partial^2 V_0}{\partial t^2} = \frac{4\pi}{c}\frac{\partial J_4}{dt} = \bar{\nabla} \cdot \frac{\partial \bar{V}}{dt} + \frac{1}{c}\frac{\partial^2 V_0}{\partial t^2}$$

while

$$\bar{\nabla} \cdot (\bar{\nabla} V_0) + \frac{1}{c}\bar{\nabla} \cdot \frac{\partial \bar{V}}{\partial t} = -a_0 \bar{\nabla} \cdot \frac{\partial \bar{E}}{\partial \gamma} = \frac{1}{c}\bar{\nabla} \cdot \frac{\partial \bar{V}}{dt} + \bar{\nabla} \cdot \bar{\nabla} V_0$$

therefore

$$\nabla^2 V_o - \frac{1}{c^2}\frac{\partial^2 V_o}{\partial t^2} = \frac{4\pi}{c^2}\frac{\partial J_4}{\partial t} - a_o\, \overline{\nabla} \cdot \frac{\partial \overline{E}}{\partial \gamma}.$$

For the vector field we have:

$$\overline{\nabla}\,(\overline{\nabla} \cdot \overline{V}) + \frac{\overline{\nabla}}{c}\frac{\partial V_o}{\partial t} = -\,\overline{\nabla}\,(\frac{4\pi}{c}\,J_4)$$

and

$$\overline{\nabla} \times (\overline{\nabla} \times \overline{V}) + \nabla^2\,\overline{V} + \frac{1}{c}\frac{\partial}{\partial t}\,\overline{\nabla}\,V_o = \frac{4\pi}{c}\,\overline{\nabla}\,J_4$$

therefore

$$\nabla^2\,\overline{V} - \frac{1}{c^2}\frac{\partial^2 \overline{V}}{\partial t^2} = \frac{4\pi}{c}\,\overline{\nabla}\,J_4 + \frac{a_o}{c}\frac{\partial^2 \overline{E}}{\partial_t \partial \gamma} + a_o\,(\overline{\nabla} \times \frac{\partial \overline{B}}{\partial \gamma}).$$

But $\overline{\nabla} \cdot \overline{E} = 4\pi\rho - a_o \dfrac{\partial V_o}{\partial \gamma}$ so that

$$\nabla^2\,V_o - \frac{1}{c^2}\frac{\partial^2 V_o}{\partial t^2} = \frac{4\pi}{c^2}\frac{\partial J_4}{\partial t} - a_o\,\frac{\partial}{\partial \gamma}\,4\pi\rho - a_o\,\frac{\partial V_o}{\partial t}$$

and $\overline{\nabla} \times \overline{B} - \dfrac{1}{c}\dfrac{\partial \overline{E}}{\partial t} = \dfrac{4\pi}{c}\overline{J} - a_o\,\dfrac{\partial \overline{V}}{\partial \gamma}$ so that

$$\nabla^2 \overline{V} - \frac{1}{c^2}\frac{\partial^2 \overline{V}}{\partial t^2} = \frac{4\pi}{c}\,\overline{\nabla}\,J_4 + a_o\,\frac{\partial}{\partial \gamma}\frac{4\pi}{c}\,\overline{J} + 2\,\frac{\partial \overline{E}}{\partial t} - a_o\,\frac{\partial \overline{V}}{\partial \gamma}.$$

Now since the wave equations for the usual vector and scalar potentials are

$$\nabla^2 \overline{A} - \frac{1}{c^2}\frac{\partial^2 \overline{A}}{\partial t^2} = -\,\frac{4\pi}{c}\,\overline{J}$$

$$\nabla^2 \phi - \frac{1}{c^2}\frac{\partial^2 \phi}{\partial t^2} = -\,4\pi\rho.$$

We may differentiate these with respect to the mass density and substitute them into our wave equations and get

$$\nabla^2\,V_o - \frac{1}{c^2}\,\frac{\partial^2}{\partial t^2}\,V_o = \frac{4\pi}{c^2}\frac{\partial J_4}{\partial t} + a_o{}^2\,\frac{\partial^2 V_o}{\partial \gamma^2}$$

(40)

$$\nabla^2\,\overline{V} - \frac{1}{c^2}\frac{\partial^2 \overline{V}}{\partial t^2} = \frac{4\pi}{c}\,\overline{\nabla}\,J_4 + a_o\,\frac{\partial}{\partial \gamma}\,[2\,\frac{\partial \overline{E}}{\partial t} - a_o\,\frac{\partial \overline{V}}{\partial \gamma}]$$

where $\mathcal{V}_0 \equiv V_0 + a_0 \frac{\partial \Phi}{\partial \gamma}$ and $\overline{\mathcal{V}} = \overline{V} - a_0 \frac{\partial \overline{A}}{\partial \gamma}$.

3. Interpretation of the Current Densities

For our system the conservation of charge becomes

$$\frac{\partial J_i}{\partial x^i} = 0; \quad i = 0, 1, 2, 3, 4$$

so that

$$\frac{\partial \rho}{\partial t} + \overline{\nabla} \cdot \overline{J} + a_0 \frac{\partial J_4}{\partial \gamma} = 0. \tag{41}$$

Thus we see that defining the current densities by equations (38) leads us to consider the new component of current density J_4 which alters the conservation of charge equation, (41).

Since equation (38) defining the current densities involves an interpretation linking these equations to reality there seems to be no *a priori* reason for this defining relationship. Defining the current density in this manner introduces also the necessity of interpreting the new term J_4, which in turn requires changing our concept of conservation of charge to that of equation (41). While the extension to five dimensions may well require changing our concept of conservation of charge, just as the step from three to four dimensions required a change in the conservation of mass, it should be possible to appeal to experimentation to determine this requirement.

Suppose we look at the defining relations

$$\frac{\partial F_{ij}}{\partial x^i} \equiv 0 \tag{42}$$

then equation (37) becomes

$$\overline{\nabla} \cdot \overline{E} = - a_0 \frac{\partial \mathcal{V}_0}{\partial \gamma} \qquad \overline{\nabla} \times \overline{B} - \frac{1}{c}\frac{\partial \overline{E}}{\partial t} = - a_0 \frac{\partial \overline{\mathcal{V}}}{\partial \gamma}$$
$$\overline{\nabla} \cdot \overline{\mathcal{V}} + \frac{1}{c}\frac{\partial \mathcal{V}_0}{\partial t} = 0. \tag{43}$$

So that we may define the charge density as

$$\rho \equiv -\frac{a_0}{4\pi}\frac{\partial V_0}{\partial \gamma} \tag{44}$$

and the current density as

$$\bar{J} \equiv -\frac{a_0 c}{4\pi}\frac{\partial \bar{V}}{\partial \gamma}. \tag{45}$$

Substituting equations (44) and (45) into the remaining equation (43) we obtain

$$\bar{\nabla} \cdot \bar{J} + \frac{\partial \rho}{\partial t} = 0 \tag{46}$$

which is the classical conservation of charge equation.

Thus if we use the defining equation (36) we are faced with interpreting the new term J_4 which has its origin in the thermodynamic variables our system. While if we choose the defining relations (42), (44), and (45) we may keep our concept of conservation of charge but this requires us, by equations (44) and (45), to consider current densities to have their origin in the thermodynamics of our system.

C. QUANTIZATION IN FIVE-DIMENSIONS

1. Quantizations

The system under consideration now is a five-dimensional system with arc element

$$(dq^0)^2 = f\,(d\sigma)^2.$$

Now since our system is a Q-conservative, $\bar{d}Q = 0$, system the principle of increasing entropy requires that $(dq^0)^2 \geq 0$ so that $f\,(d\sigma)^2 \geq 0$. Introducing the quantitization conditions results in

$$\int \phi_j\, dx^j = 2\pi i n; \quad j = 0, 1, 2, 3, 4$$

where $\phi_j \equiv \pm \frac{\partial \ln f^{1/2}}{\partial x^j}$ and $x^0 \equiv ct$; $x^1 \equiv q^1$; $x^2 \equiv q^2$; $x^3 \equiv q^3$; $x^4 \equiv \frac{\gamma}{a_0}$.

If we restrict ourselves to a $(d\sigma)^2$ space which is the local Euclidean space then $(d\sigma)^2$ is the five-dimensional Minkowski-type manifold and using London's work we would produce a five-dimensional quantum dynamical system.

2. Five-Dimensional Hamiltonian

We previously showed that the principle of increasing entropy resulted in

$$\delta \int \gamma \sqrt{(dq^o)^2} = 0$$

as the variational principle for a local Euclidean manifold. Since multiplication by a constant does not change the problem we may take our variational problem to be

$$\delta \int \gamma \; c^2 \sqrt{(dq^o)^2} = 0.$$

Defining the velocity vector as $u^j \equiv \frac{dx^j}{dq^o}$ and the momentum as $p_j \equiv \frac{\partial L}{\partial u^j} \equiv \gamma \; g_{jk}u^k$, where we have used the fact that $q_{jk}u^ju^k = 1$, then we may form the contravariant momentum as

$$p^j = g^{jk}p_k = g^{jk} \gamma \; g_{k\ell} \; u^\ell$$

so that

$$\begin{aligned} p_jp^j &= (\gamma \; g_{j\ell}u^j) \; (g^{jk} \gamma \; g_{k\ell} \; u^\ell) = \gamma^2 \; \delta_{j\ell} \; u^\ell \; g_{jk} \; u^k \\ &= \gamma \; (\gamma \; g_{jk} \; u^ju^k) \\ &= \gamma^2c^2, \end{aligned} \tag{47}$$

since $\gamma c^2 = \gamma \; g_{jk}u^ju^k$. Equation (47) is the five-dimensional "momentum energy" equation.

We may now follow London's procedure to obtain our wave function for the five-dimensional system. However a quicker way to investigate the effect of the Dynamic Theory upon quantum mechanics would seem to be that of adopting Dirac's equation in a five-dimensional form and following a development analogous to standard four-dimensional relativistic quantum mechanics. With this in mind then we shall adopt the form

$$h = i\ (\alpha_1 \frac{\partial}{\partial x^1} + \alpha_2 \frac{\partial}{\partial x^2} + \alpha_3 \frac{\partial}{\partial x^3} + \alpha_4 \frac{\partial}{\partial x^4}) - \beta \tag{48}$$

to be the five-dimensional specific Hamiltonian operator. In equation (48) the α's and β do not involve derivatives and must be Hermitian in order that h be Hermitian.

By taking the four partial derivatives in equation (48) as the 4-vector momentum operator we may write

$$h = -\ (\overline{\alpha} \cdot \overline{p} + \beta). \tag{49}$$

3. Five-Dimensional Dirac Equation

If we take $p^0 \mid > = h \mid >$ and require that the α's and β are chosen such that solutions of this equation are also solutions of equation (47) we find the restrictions imposed upon the choice of the α's and β to be:

$$\begin{aligned} (\overline{\alpha} \cdot \overline{p})^2 &= p^2 \\ \beta^2 &= 1 \\ \overline{\alpha}\beta + \beta\overline{\alpha} &= 0 \end{aligned} \tag{50}$$

where natural units, c = 1, are used.

A set of 4 x 4 matrices satisfying the requirements of equation (50) is

$$\beta = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix};\ \alpha_j = \begin{pmatrix} 0 & \sigma_j \\ \sigma_j & 0 \end{pmatrix},\ j = 1, 2, 3; \quad \alpha_4\ \begin{pmatrix} -\sigma_2 & 0 \\ 0 & -\sigma_2 \end{pmatrix} \tag{51}$$

where I is the 2 x 2 identity matrix and the σ's are the 2 x 2 Pauli spin matrices.

Then the five-dimensional Dirac equation may be taken to be

$$i \frac{\partial}{\partial t} \Psi (x) = (i \bar{\alpha} \cdot \nabla - \beta) \Psi (x) \tag{52}$$

where the ∇ is a four-dimensional operator. By defining

$$\gamma^0 \equiv \beta \; ; \quad \gamma^\mu \equiv - \beta\alpha_\mu \; (\mu = 1, 2, 3, 4) \tag{53}$$

then equation (52) may be written as

$$(i \, \partial_j \gamma^j + 1) \Psi (x) = 0. \tag{54}$$

By virtue of the properties of the α's and β plus the fact that

$$g^{jk} = \begin{cases} 1 \text{ for } j = k = 0 \\ -1 \text{ for } j = k = k, 2, 3, 4 \\ 0 \text{ for } j \neq k \end{cases}$$

the anticommutator of the γ-matrices must satify

$$\{\gamma^j, \gamma^i\} = 2g^{ji}.$$

In standard representation the γ-matrices are given by

$$\gamma^0 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}; \quad \gamma^\mu = \begin{pmatrix} 0 & -\sigma_\mu \\ \sigma_\mu & 0 \end{pmatrix}, \quad \mu = 1, 2, 3; \; \gamma^4 = \begin{pmatrix} -\sigma_2 & 0 \\ 0^2 & \sigma_2 \end{pmatrix}.$$

4. "Lorentz" Covariance

Under a five-dimensional Lorentz transformation

$$x^{j'} = L^j{}_k \, x^\varepsilon$$

we shall suppose each component of the wave function $\Psi (x)$ transforms into a linear combination of all four components;

$$\Psi (x) \overrightarrow{LT} \Psi' (x') = S \Psi (x) \tag{55}$$

where S is a Dirac spinor satisfying

$$S^{-1} \gamma^j S = L^j{}_k \gamma^k. \tag{56}$$

By using an infinitesimal Lorentz transformation given by

$$L^j{}_k = g^j{}_k + d\theta \varepsilon^j{}_k$$

where $\varepsilon^j{}_k$ are a set of 16 numbers, then $S(\theta)$ may be shown to be given by

$$S(\theta) = \exp \left(T \int_0^\theta d\theta\right) \tag{57}$$

where the matrix T is given by

$$T = \frac{1}{4} \varepsilon_{jk} \gamma^j \gamma^k.$$

Equations (55), (56) and (57) suffice to guarantee the Lorentz covariance of the five-dimensional Dirac equation.

5. "Free Particle" Solutions

If we look for solutions of equation (55) which are also eigenfunctions of the operator $p^j = i\partial^j$ then we may write the wave function as

$$\Psi(x) = w(p)\, e^{-ip_j x^j}. \tag{58}$$

By substituting equation (58) into equation (55) we find that $w(p)$ must satisfy

$$(p_j \gamma^j + 1)\, w(p) = 0. \tag{59}$$

Using the standard representation of the γ-matrices equation (59) may be written

$$
\begin{pmatrix}
p_0 + 1 & ip_4 & -p_3 & -p_1 + ip_2 \\
-ip_4 & p_0 + 1 & -p_1 - ip_2 & p_3 \\
p_3 & p_1 - ip_2 & p_0 + 1 & -ip_4 \\
p_1 + ip_2 & -p_3 & +ip_4 & -p_0 + 1
\end{pmatrix}
\begin{pmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \end{pmatrix} = 0 \tag{60}
$$

where it is important to remember that p_1, p_2, p_3 and p_4 represent minus the respective components of $\overline{p}$. This set of four, linear, homogeneous, algebraic equations has a nontrivial solution only if the determinant of the square matrix on the left hand side vanishes. This determinant is $(p_0{}^2 - p^2 - 1)^2$. Thus equation (58) is a solution of the Dirac equation only if

$$p_0 = \pm\ (p^2 + 1)^{1/2}. \tag{61}$$

By defining $\varepsilon\ (p) = (p^2 + 1)^{1/2}$ then equation (61) becomes

$$p_0 = \pm\ \varepsilon\ (p). \tag{62}$$

Substituting equation (62) into equation (60) the solutions are found to be:

for $p_0 = +\varepsilon$

$$
u_1\ (p) = N \begin{pmatrix} \dfrac{p_3}{\varepsilon + 1} \\ \dfrac{p_1 + ip_2}{\varepsilon + 1} \\ 1 \\ \dfrac{ip_4}{\varepsilon + 1} \end{pmatrix}
\qquad
u_2\ (p) = N \begin{pmatrix} \dfrac{p_1 - ip_2}{\varepsilon + 1} \\ \dfrac{-p_3}{\varepsilon + 1} \\ \dfrac{-ip_4}{\varepsilon + 1} \\ 1 \end{pmatrix} \tag{63}
$$

for $p_o = -\varepsilon$

$$v_1(p) = N \begin{pmatrix} 1 \\ \frac{-ip_4}{\varepsilon + 1} \\ \frac{-p_3}{\varepsilon + 1} \\ \frac{-p_1 - ip_2}{\varepsilon + 1} \end{pmatrix} v_2(p) = N \begin{pmatrix} \frac{ip_4}{\varepsilon + 1} \\ 1 \\ \frac{-p_1 + ip_2}{\varepsilon + 1} \\ \frac{p_3}{\varepsilon + 1} \end{pmatrix} \tag{64}$$

where N is a constant.

Following standard quantum mechanical procedure we shall adopt the probability current density to be

$$j^k(x) = \overline{\Psi}(x)\,\gamma^k\,\Psi(x)$$

with the requirements:

(1) $\partial_k j^k = 0$

(2) j^k transforms as a contravariant vector, and

(3) j^k must be real.

We can determine the normalizing constant N by using the fact that

$$m = \int_V \gamma\, dx^3$$

then calculating the expectation value of the mass. Thus

$$< m > = \int_V \Psi^\dagger\, m\, \Psi\, d^3x$$

where $\dagger$ represents the transposed complex conjugate. Then using any of the solutions given by equations (63) or (64) the expectation value becomes

$$< m > = \frac{2N^2 m\varepsilon V}{(\varepsilon + 1)}$$

so that

$$N = \left(\frac{\varepsilon + 1}{2m\varepsilon V}\right)^{1/2}. \tag{65}$$

Thus the "free particle" solutions of the five-dimensional Dirac equation are given by equations (63) and (64) with the constant having the value given by equation (65).

6. Spin

In the three-dimensional space the angular momentum is given the vector, L, as the cross product of the coordinates and momenta. We shall then define the angular 4-momentum to be the four-dimensional cross product

$$\overline{L} \equiv \varepsilon_{ijk} x^j p^k$$

where x^4 is the mass density and

$$\varepsilon_{ijk} \equiv \begin{cases} 0 \text{ if any two indices are alike} \\ 1 \text{ for even permutation to align indices in ascending order} \\ -1 \text{ for odd permutation to align indices in ascending order.} \end{cases}$$

Then the comutator of the components of the angular Ψ-momentum with the specific Hamiltonian are not zero, for instance

$$[L_3, h] = i\gamma^0\gamma^1p^2 - i\gamma^0\gamma^2p^1 + i\gamma^0\gamma^4p^1 - i\gamma^0\gamma^1p^4 + i\gamma^0\gamma^4p^2 - i\gamma^0\gamma^2p^4.$$

Now suppose there exists a 4-spin vector $\overline{S}$ such that the sum of the angular 4-momentum and the 4-spin vector commutes with the specific Hamiltonian, then if we define a new 3-spin vector to be $\overline{u}$, given by the components $u_1 \equiv \frac{1}{2} i\gamma^4\gamma^1$, $u_2 \equiv \frac{1}{2} i\gamma^4\gamma^2$, and $u_3 \equiv \frac{1}{2} i\gamma^4\gamma^3$, and take the usual spin vector, $\overline{s}$, given by $s_1 = \frac{1}{2} i\gamma^2\gamma^3$, $s_2 = \frac{1}{2} i\gamma^3\gamma^1$, and $s_3 = \frac{1}{2} i\gamma^1\gamma^2$, the components of the 4-spin vector may be shown to be

$$S_1 = s_1 - u_2 - u_3$$

$$S_2 = s_2 + u_1 - u_3$$

$$S_3 = s_3 + u_1 - u_2$$

$$S_4 = s_1 - s_2 + s_3$$

In analogy with standard relativistic quantum mechanics the eigen values of the 4-spin components can be shown to be $\pm \sqrt{\frac{3}{4}}$. It may also be shown that the set of observables $\overline{P}$, h, and $\overline{S} \cdot \overline{P}$, where $\overline{P}$ is the 4-momentum and $\overline{S}$ is the 4-spin, form a complete set of commuting observables.

7. Dirac Equation with Fields

In analogy with relativistic quantum mechanics we take the 5-dimensional Dirac equation to be

$$[(i\partial_j - \phi_j)\,\gamma^i + 1]\,\Psi = 0 \tag{66}$$

where ϕ_j is 5-vector potential.
By operating on the left with $[(i\partial_j - \phi_j)\gamma^j - 1]$ and separating $\gamma^j\gamma^k$ into symmetric and anti symmetric parts as

$$\gamma^i\gamma^k = \frac{1}{2}\{\gamma^i,\gamma^k\} + \frac{1}{2}[\gamma^j,\gamma^k] \equiv g^{ik} + \sigma^{jk} \tag{67}$$

then equation (66) becomes

$$[(i\partial_j - \phi_j)(i\partial^j - \phi^j) - 1 + (-\partial_j\partial_k + \phi_j\phi_k - i\phi_j\partial_k - i\partial_j\phi_k)\,\sigma^{jk}]\,\Psi = 0. \tag{68}$$

Separating $\partial_j\phi_k$ into symmetric and anti symmetric parts as

$$\partial_j\phi_k = \frac{1}{2}(\partial_j\phi_k + \partial_k\phi_j) + \frac{1}{2}(\partial_j\phi_k - \partial_k\phi_j)$$

and defining the field tensor as

$$F_{jk} = \partial_j\phi_k - \partial_k\phi_j$$

then equation (68) becomes

$$[(i\partial_j - \phi_j)(i\partial^k - \phi^k) - 1 - \frac{1}{2}\, iF_{jk}\sigma^{jk}]\,\Psi = 0. \tag{69}$$

Now since

$$\sigma^{jk} = \begin{pmatrix} 0 & \dot{x}^1 & \dot{x}^2 & \dot{x}^3 & \dot{x}^4 \\ -\dot{x}^1 & 0 & -2is^3 & 2is^2 & n^1 \\ -\dot{x}^2 & 2is^3 & 0 & -2is^1 & n^2 \\ -\dot{x}^3 & -2is^2 & 2is^1 & 0 & n^3 \\ -\dot{x}^4 & -n^1 & -n^2 & -n^3 & 0 \end{pmatrix}$$

where $\sigma^{j4} \equiv n^j$ for $j = 1, 2, 3$, and

$$F_{jk} = \begin{pmatrix} 0 & E_1 & E_2 & E_3 & V_0 \\ -E_1 & 0 & -B_3 & B_2 & V_1 \\ -E_2 & B_3 & 0 & -B_1 & V_2 \\ -E_3 & -B_2 & B_1 & 0 & V_3 \\ -V_0 & -V_1 & -V_2 & -V_3 & 0 \end{pmatrix}$$

plus recalling the seven Maxwell-type equations

$$\begin{aligned} &\bar{\nabla} \cdot \bar{B} = 0 && \bar{\nabla} \times \bar{E} + \frac{1}{c}\frac{\partial \bar{B}}{\partial t} = 0 \\ &\bar{\nabla} \cdot \bar{E} = 4\pi\rho - a_0 \frac{\partial V_0}{\partial \gamma} && \bar{\nabla} \times \bar{B} - \frac{1}{c}\frac{\partial \bar{E}}{\partial t} = \frac{4\pi \bar{J}}{c} - a_0 \frac{\partial \bar{V}}{\partial \gamma} \\ &\bar{\nabla} \times \bar{V} + a_0 \frac{\partial \bar{B}}{\partial \gamma} = 0 && \bar{\nabla} V_0 + \frac{1}{c}\frac{\partial \bar{V}}{\partial t} + a_0 \frac{\partial \bar{E}}{\partial \gamma} = 0 \end{aligned} \tag{70}$$

$$\bar{\nabla} \cdot \bar{V} + \frac{1}{c}\frac{\partial V_0}{\partial t} = -\frac{4\pi}{c} J_4.$$

Then equation (69) may be written as

$$[(i\partial_j - \phi_j)(i\partial^k - \phi^k) - 1 + 2\bar{B} \cdot \bar{s} - i\bar{E} \cdot \dot{\bar{x}} - iV_0\dot{x}^4 - i\bar{n} \cdot \bar{V}]\,\Psi = 0 \tag{71}$$

and thus becomes the Dirac equation with fields $\bar{E}$, $\bar{B}$, V_0, and $\bar{V}$.

Suppose we consider a system without an electric charge so that $p = \overline{J} = 0$, then by equation (70) we still have

$$\overline{\nabla} \cdot \overline{E} = - a_o \frac{\partial V_o}{\partial \gamma} \quad \text{and} \quad \overline{\nabla} \times \overline{B} - \frac{1}{c}\frac{\partial \overline{E}}{\partial t} = - a_o \frac{\partial \overline{V}}{\partial \gamma} \tag{72}$$

and therefore there will still be a magnetic moment.

D. SUMMARY

If indeed the Dynamic Theory corresponds to reality then any system which requires thermodynamic as well as mechanical variables for its description must be described in a five-dimensional manifold of entropy-space-mass. A Q-conservative system which is very near an equilibrium state must be described in a five-dimensional manifold of space-time-mass. Thus, for such a system, mass enjoys a status equivalent to that of space and time in the Special Theory of Relativity.

When the system is taken to be far enough from an equilibrium state for the second partial derivatives to be taken as functions, the Dynamic Theory readily yields a variational principle from the field equations and equations of motion may be determined.

The seven Maxwell-type equations, equations (37) and (38) introduce four new field quantities that form an interrelationship with the electromagnetic field. In Section III.B.4 these quantities pose a question of interpretation. However, these field quantities may be seen from a different perspective in the following sections.

The formulism of standard relativistic quantum mechanics was carried out in the five-dimensional manifold of the Dynamic Theory. It seems significant, or at least interesting, to note that the effect of the fifth dimension is to fill the zeros normally found in the square matrix of $(p_j \gamma^j + 1)$ of equation (60).

With the development of the spin and the five-dimensional Dirac equation with fields comes the possibility of finding an interpretation of the new field quantities. This possibility may be seen in the following argument.

Suppose that an electron, because of its small amount of mass when compared to a proton or neutron, does not involve sufficient mass density change to warrant using the 5th dimension. Then the magnetic moment of the electron should be given accurately by the four-dimensional Dirac equation. The accuracy of these predictions is already known.

But suppose that nucleons (i.e., protons and neutrons) have sufficient mass density change to warrant using the 5th dimension provided by the Dynamic Theory. Then the nucleons should be described by equation (69). Using equation (70) we should expect a different value for the magnetic moment of a proton than the prediction of the four-dimensional Dirac equation since the field equations involve additional source terms. For the neutron, which has no electric charge, we would also find a magnetic moment predicted by equation (69) because of these additional source terms.

Now if we assume that the differences between the observed values of the magnetic moments of the proton and neutron and the predicted values of relativistic quantum mechanics are due to the strong interaction or nuclear forces, then we must connect the new terms in equation (70) with the nuclear charge and nuclear current densities. Then the new field quantities may play the dominant role in the realm of nuclear physics. Then the field equation (70), plus equation (71) and the three spin vectors, $\overline{S}$, $\overline{s}$ and $\overline{u}$ may provide answers to questions in nuclear and elementary particle physics.

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